Operational Amplifier Stability
Part 3 of 15: R_O and R_{OUT}

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Part 3 focuses on clarifying some common misconceptions regarding op amp "Output Resistance." We define two important, different, output resistances: R_O and R_{OUT}. R_O will become extremely useful when we start to stabilize op amp circuits that are driving capacitive loads. We present easy techniques to derive R_O from op amp manufacturers' data sheets and, in addition, a couple of real-world measurement techniques for those op amps whose data sheets do not contain a specification for R_O. We also show a trick for using SPICE op amp models and R_O which will allow you to use the SPICE loop-gain test while including the effects of R_O.

Definition and Derivation of R_O and R_{OUT}

R_O is defined in this series as the open-loop output resistance of an op amp while R_{OUT} is defined as the closed-loop output resistance. Fig. 3.0 emphasizes the important difference between these two.

R_O = Op Amp ***Open Loop*** Output Resistance
R_{OUT} = Op Amp ***Closed Loop*** Output Resistance

Fig. 3.0: Definition of R_O and R_{OUT}

As hinted at in Fig. 3.0, R_O and R_{OUT} are related: R_{OUT} is R_O reduced by loop gain. Fig. 3.1 defines the simplified op amp model used for the derivation of R_{OUT} from R_O, focusing solely on the basic dc characteristics. A high input resistance, R_{DIFF},(100 MΩ to GΩ) is between –IN and +IN and the voltage difference between them develops an error voltage, V_E, across it which is amplified by the open-loop gain, Aol, and becomes V_O. In series with it to the output, V_{OUT}, is R_O, the open-loop output resistance.

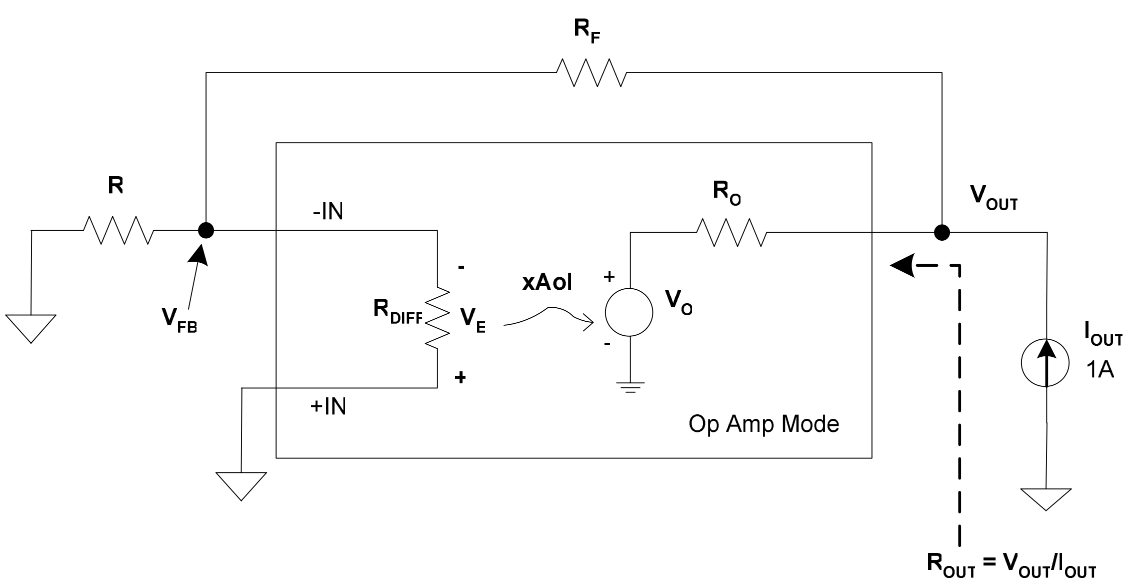


Fig. 3.1: Op Amp Model For Derivation of R_{OUT}

Using the op amp model in Fig. 3.1 we can solve for R_{OUT} as a function of R_O and Aolβ. This derivation is detailed in Fig. 3.2. We see that Aolβ, loop gain, reduces R_O so that the output resistance of the op amp with feedback, R_{OUT}, will be much lower than R_O, for large values of Aolβ.

1) $\beta = V_{FB} / V_{OUT} = [V_{OUT} (R_I / \{R_F + R_I\})]/V_{OUT} = R_I / (R_F + R_I)$

2) $R_{OUT} = V_{OUT} / I_{OUT}$

3) $V_O = -V_E \, Aol$

4) $V_E = V_{OUT} [R_I / (R_F + R_I)]$

5) $V_{OUT} = V_O + I_{OUT}R_O$

6) $V_{OUT} = -V_E Aol + I_{OUT}R_O$ *Substitute 3) into 5) for* V_O

7) $V_{OUT} = -V_{OUT} [R_I/(R_F + R_I)] \, Aol + I_{OUT}R_O$ *Substitute 4) into 6) for* V_E

8) $V_{OUT} + V_{OUT} [R_I/(R_F + R_I)] \, Aol = I_{OUT}R_O$ *Rearrange 7) to get* V_{OUT} *terms on left*

9) $V_{OUT} = I_{OUT}R_O / \{1+[R_I Aol/(R_F+R_I)]\}$ *Divide in 8) to get* V_{OUT} *on left*

10) $R_{OUT} = V_{OUT}/I_{OUT} =[I_{OUT}R_O / \{1+[R_I Aol / (R_F+R_I)]\}] / I_{OUT}$

Divide both sides of 9) by I_{OUT} *to get* R_{OUT} *[from 2)] on left*

11) $R_{OUT} = R_O / (1+Aol\beta)$ *Substitute 1) into 10)*

Fig. 3.2: Derivation of R_{OUT}

Computing R_O From Data Sheet Curves

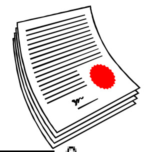
The OPA353 is a wideband (UGBW = 44 MHz, SR = 22 V/µs, settle to 0.1% = 0.1 µs) CMOS, single supply (2.7 V to 5.5 V), RRIO (rail-to-rail input and output) op amp. There is no R_O specification in the table of specifications in the data sheet. However, there are two helpful curves to help us determine R_O. We will need to use the open-loop gain/phase vs frequency curve (see Fig. 3.3) and the closed-loop output impedance vs frequency curve (see Fig. 3.4) to easily calculate R_O. The closed-loop output impedance vs frequency curve is actually a plot of R_{OUT} vs frequency.

Within the unity-gain bandwidth of voltage-feedback op amps, R_O and R_{OUT} are predominantly resistive. On the closed-loop output impedance vs frequency curve, Fig. 3.4, we choose the G = 10 curve and on its x-axis the point 1 MHz (just choose an easy to read data point). At the intersection of 1 MHz and G = 10 curve we see R_{OUT} = 10 Ω.

On the open-loop gain/phase vs frequency curve, Fig. 3.3, we look at the 1-MHz frequency point on the x-axis and read the open-loop gain as 29.54 dB (We measured this one with a ruler and scaled it based on the linear dB y-axis. We did this on an enlarged cut-and-paste curve). The derivation of R_O from the information collected in Figs. 3.3 and 3.4 is detailed in Fig. 3.5. Now, from our formula for R_O we rearrange the equation to give us R_O in terms of R_{OUT}, Aol, and β. From this equation and our data sheet information we calculate the R_O for the OPA353 to be 40 Ω.

OPA353 Specifications:

Aol @1MHz = 29.54dB = x30



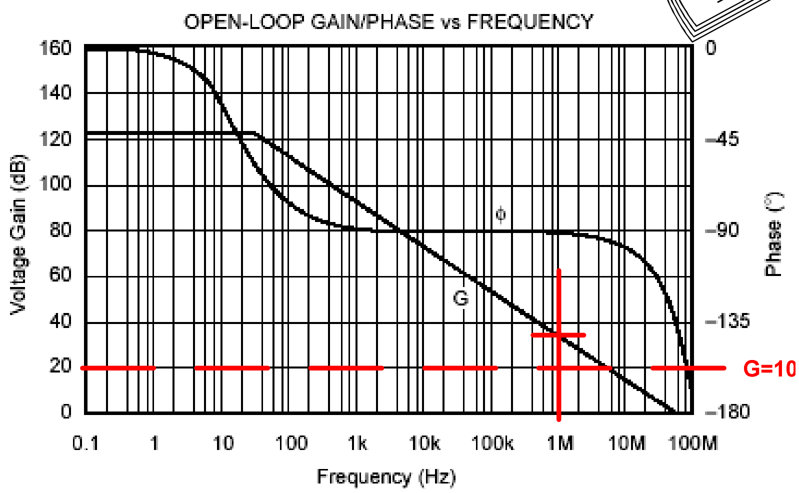
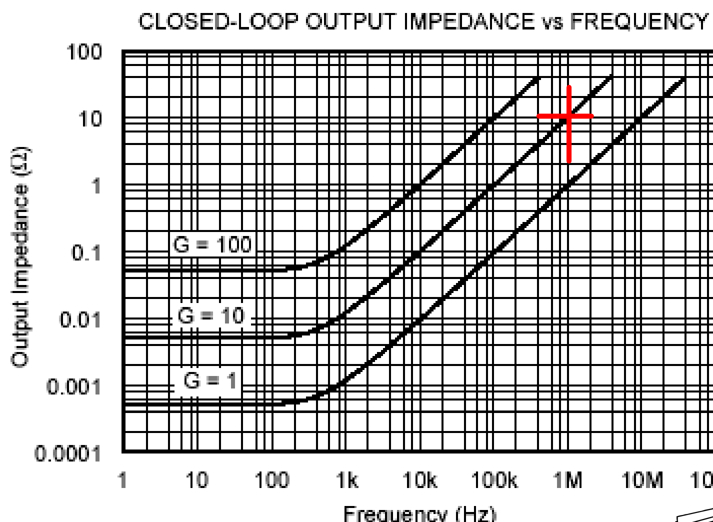


Fig. 3.3: OPA353 Aol Vs Frequency



OPA353 Specifications:

R_{OUT} (@1MHz, G=10) = 10Ω

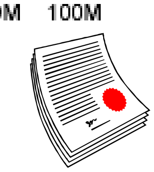


Fig. 3.4: OPA353 Closed-Loop Output Impedance Vs Frequency

OPA353 R_O Calculation

$R_{OUT} = R_O / (1+Aol\beta)$

$R_O = R_{OUT} (1+Aol\beta)$

$R_O = 10\Omega\ (1+ 30[1/10])$

$R_O = 40\Omega$

Fig. 3.5: OPA353 RO Calculation

We can use the op amp model (Fig. 3.1) and the information from the OPA353 data sheet to fill in actual values in the model (see Fig 3.6) and see how our model correlates with real-world op amps. Notice in this model we define V_O as the op amp's output before R_O, and V_{OUT} as the actual op amp output. Of course in a real op amp we can only gain access to V_{OUT} but this model and the fact that we can get real world data to build this model will become very powerful in stability analysis.

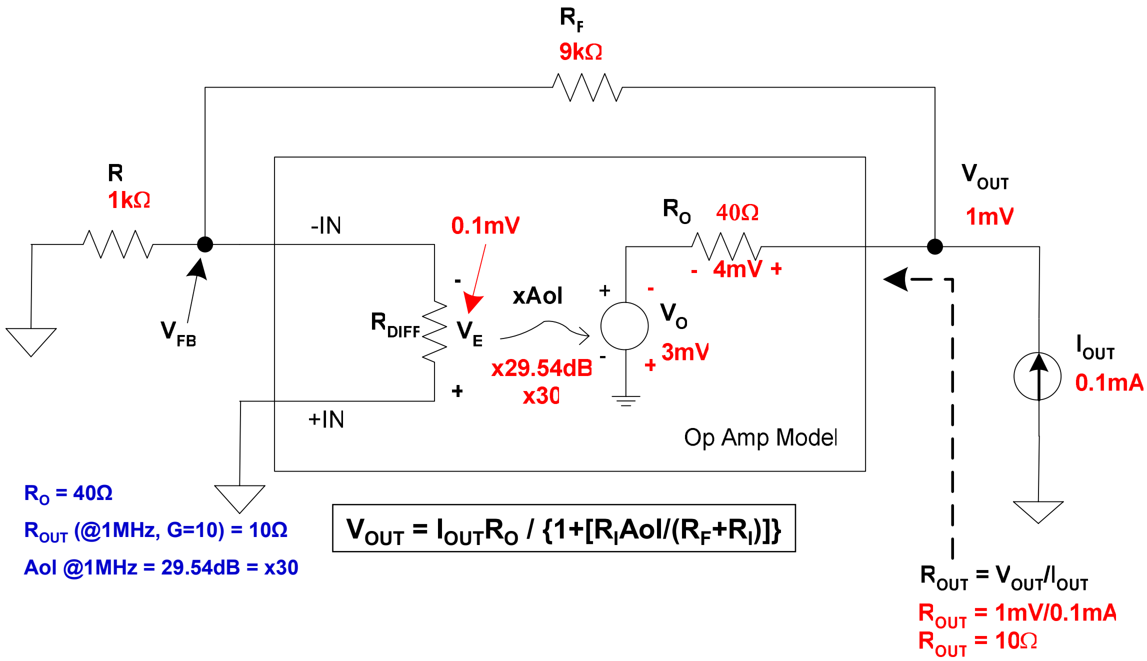


Fig. 3.6: OPA353 R_O Calculation Using Op Amp Model

Summary of R_O and R_{OUT} Key Points

- **R_O does *NOT* change when Closed Loop feedback is used**
- **R_{OUT} is the effect of R_O, Aol, and β controlling V_O**
  - ✓ **Closed Loop feedback (β) forces V_O to increase or decrease as needed to accommodate V_O loading**
  - ✓ **Closed Loop (β) increase or decrease in V_O appears at V_{OUT} as a reduction in R_O**
  - ✓ **R_{OUT} increases as Loop Gain (Aolβ) decreases**

Fig. 3.7: R_O Vs R_{OUT}

- ***R_O is constant over the Op Amp's bandwidth*** 
- ***R_O is defined as the Op Amp's Open Loop Output Resistance***
- ***R_O is measured at I_{OUT} = 0 Amps, f = 1MHz***
  (use the unloaded R_O for Loop Stability calculations since it will be the largest value → worst case for Loop Stability analysis)
- ***R_O is included when calculating β for Loop Stability analysis***

Fig. 3.8: R_O Key Points

R_O and SPICE Simulations

Fig. 3.9 shows a simple ac SPICE model for the OPA353 and we use the 40 Ω computed for R_O. Notice that we break the loop for ac stability analysis using the SPICE loop-gain test between R_O and V_O to analyze the effects of R_O on $1/\beta$. This will become extremely important in stabilizing capacitive loads driven by op amps (which will be covered in detail in Parts 7 and 8 of this series).

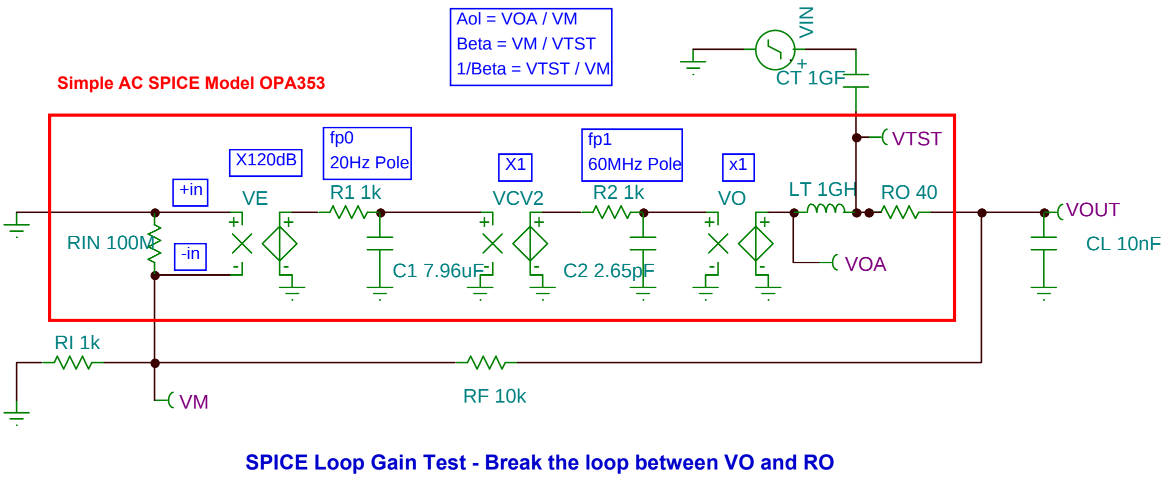
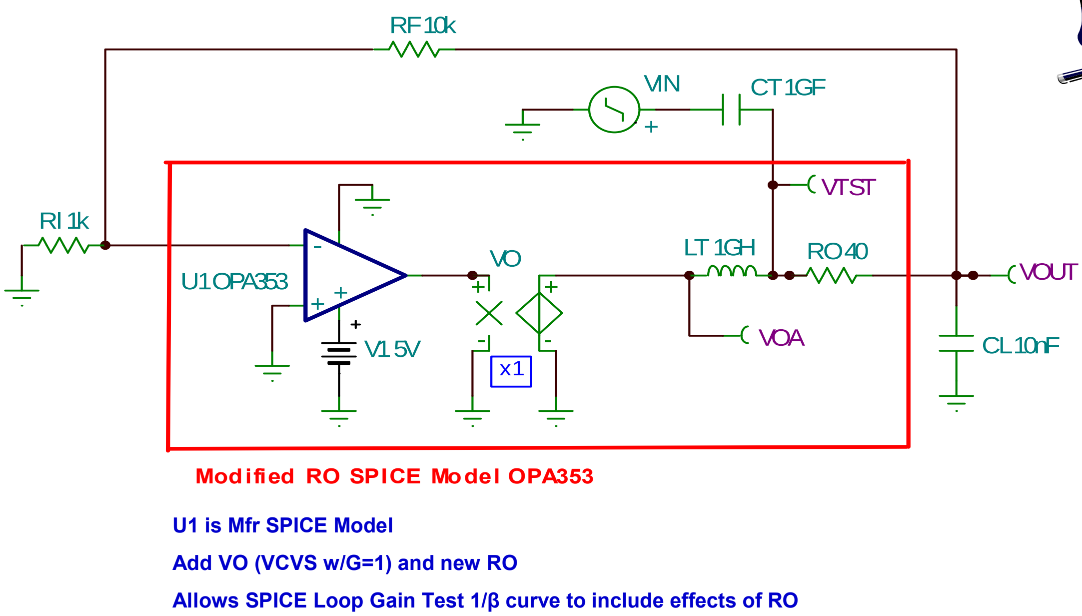


Fig. 3.9: Simple Ac SPICE Model With R_O



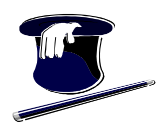


Fig. 3.10: Modified R_O SPICE Macromodel

For an existing op amp SPICE model we can easily add external R_O so that when we use the SPICE loop gain test to find $1/\beta$ we can include the effects of R_O. In the Modified R_O SPICE model in Fig. 3.10 we add a voltage-controlled voltage source (VCVS), V_O, with a gain of one. This isolates the op amp's output and any internal R_O it may have modeled from whatever connects to VOA. Now we can add our own R_O after V_O, and break the loop between V_O and R_O -- which is desired for including the effects of R_O when analyzing capacitive loads and their effects on $1/\beta$.

Real World R_O for Single-Supply Op Amps

Fig.3.11 lists some real-world measured R_O for a number of single-supply op amps. Notice that the OPA353 we analyzed to be $R_O = 40\ \Omega$ has a measured value of 44 Ω. This close correlation is because the data we used from the manufacturer's data sheet was also measured data on a typical part!

Part	R_O (ohms)	Part	Ro (ohms)	Part	Ro (ohms)
OPA132	80	OPA348	600	OPA627	55
OPA227	40	OPA350	50	OPA684	50
OPA277	10	OPA353	44	THS4503	14
OPA300	20	OPA354	35	TLC080	100
OPA335	90	OPA355	40	TLC081	100
OPA336	250	OPA356	30	TLC2272	140
OPA340	80	OPA363	160	TLE2071	80
OPA343	80	OPA380	30	TLV2461	173

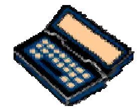
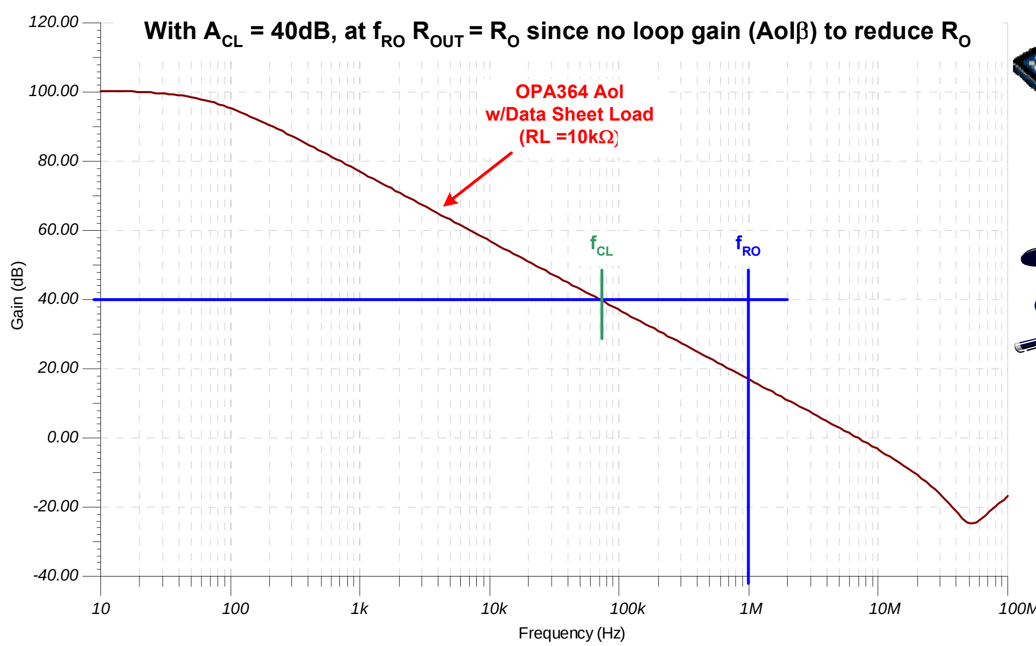
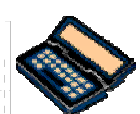


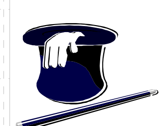
Fig. 3.11: Real World R_O For Some Single-Supply Op Amps

Real World Measurement Techniques for R_O

So what if we do not have any manufacturer's specifications for R_O and we want to know what it is? There are two real world techniques we can use and each starts by looking at the open-loop gain/phase vs frequency curve. Such a curve is shown in Fig. 3.12 for the OPA364, a wideband CMOS, single supply, RRIO op amp with "linear offset over common-mode range" but if we choose to test this op amp at a gain of 100 and at 1 MHz there will be no loop gain, Aolβ, left. Therefore, if we measure R_{OUT} under these conditions we will really be getting a value for R_O!







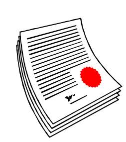


Fig. 3.12: Trick For Measuring R_O

The test circuit in Fig. 3.13 shows one method for measuring R_O in the real world, which we will call R_O–drive. Here, the output of an OPA364 passes through an ac coupling capacitor, C1, to ensure we do not load down the amplifier with any dc. Most op amp R_Os gets smaller as large currents are driven through them. We want to measure R_O at its highest value (which will cause the most problems during ac stability analysis) and here the voltage at the output of the amplifier, V_O, is measured as is the voltage, VTest, at the junction of the ac coupling capacitor, C1, and the current limiting resistor, R3. The current into the op amp's output is calculated and used to divide the voltage at the op amp to give us the measured R_O value! Note that although the OPA364 is a single-supply op amp we can cleverly run it at +2.5 V and -2.5 V to avoid a more complicated level-shifting of our input or output signal.

NOTE: All measurements used in this drive method must be ac with no dc component. If you use the "ac analysis/calculate nodal voltages" in TINA SPICE you will get an rms voltage reading at the nodes which includes the dc voltages in the circuit (ie offset referred-to-output). If the offset voltage is significant in comparison to the ac voltage components then an erroneous R_O will be calculated! In Fig. 3.13 the ac analysis/calculate nodal voltages was used but the dc offset at VOA is about 87.63 µV in comparison to 34.87 mV and 353.55 mV rms values which are dominated by ac voltage components.

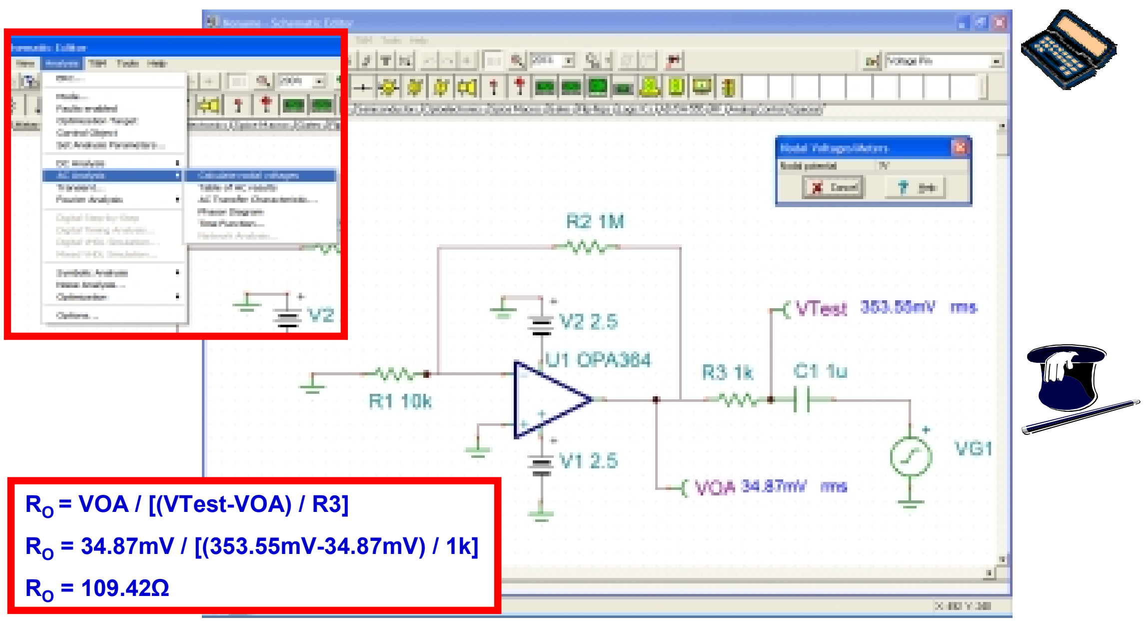
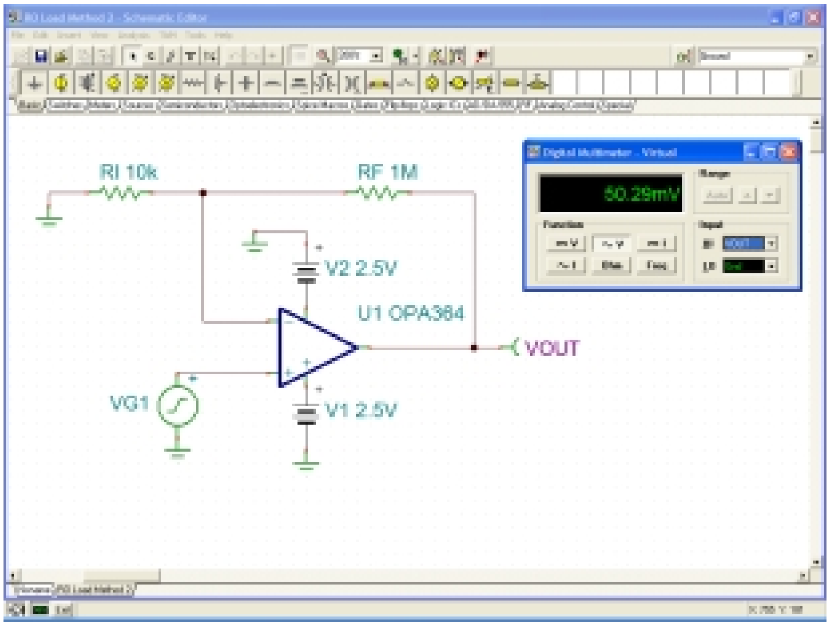
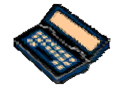


Fig. 3.13: Measuring R_O–Drive Method

The test circuits in Figs. 3.14 and 3.15 show another method for measuring R_O in the real world. This technique takes a voltage reading out of the op amp both loaded and unloaded and then computes the value for R_O. We still need to use a high gain and frequency combination to ensure there is no loop gain reducing R_{OUT} for our measurements. In this configuration a small ac signal is injected into the op amp's input. Both inverting or non-inverting gain will work. In Fig. 3.14 we measure V_{OUT}, the unloaded voltage -- a small value that will not pull much current.

NOTE: All measurements used in the load method must be ac with no dc component. If the "ac analysis/calculate nodal voltages" in TINA SPICE is used you will get an rms voltage reading at the nodes that includes the dc voltages in the circuit (ie offset referred-to-output). If the offset voltage is significant in comparison to the ac voltage components then an erroneous R_O will be calculated!





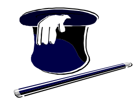
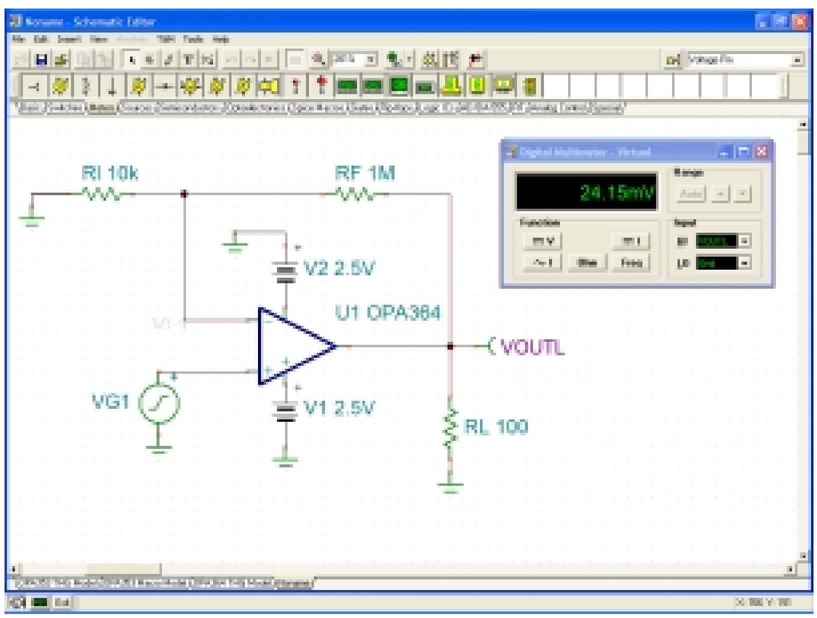


Fig. 3.14: Measuring R_O–Load Method, V_{OUT} Unloaded

In Fig. 3.15 we measure V_{OUTL}, the loaded value of V_{OUT} when R_L is attached to the output of the op amp. Note how the value of R_L does not cause large currents to flow into or out of the op amp's output.



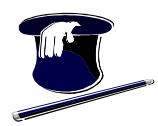


Fig. 3.15: Measuring R_O–Load Method, V_{OUT} Loaded

Now we have completed our measurements for the R_O-load method a simple calculation will result in the value for R_O. The unloaded value, V_{OUT}, is always there at V_O whether or not a load, R_L, is present. From this we can create the final model shown in Fig. 3.16. I_{OUT}, is, by inspection just V_{OUTL}/R_L. The drop across R_O is V_{OUT}-V_{OUTL} and divided by the current through it will give us the value for R_O. This method yields R_O = 108.2 Ω and the R_O-drive method yielded R_O = 109.42 Ω. Either method is acceptable to measure real-world R_O.

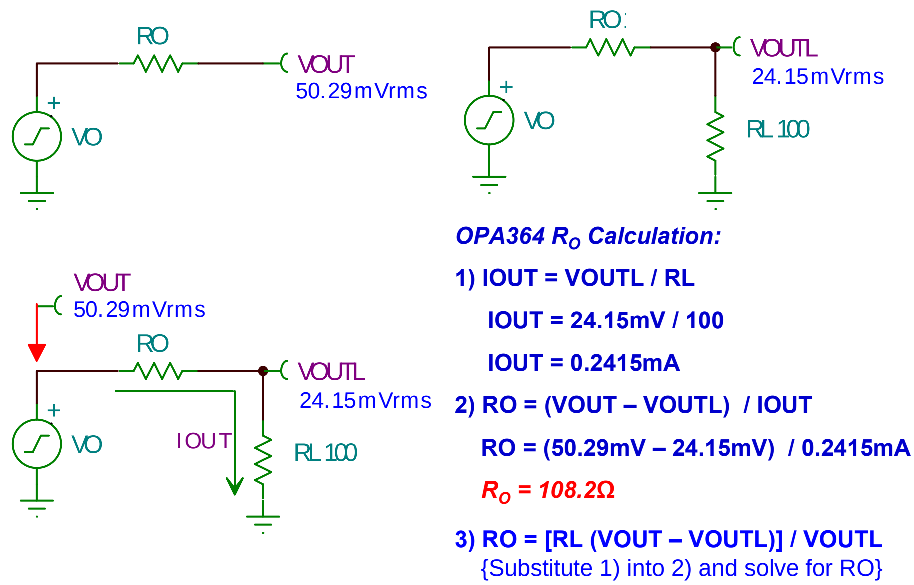


Fig. 3.16: Measuring R_O–Load Method Calculation

Reference

Frederiksen, Thomas M. Intuitive Operational Amplifiers, From Basics to Useful Applications, Revised Edition. McGraw-Hill Book Company. New York, New York. 1988

About The Author

After earning a BSEE from the University of Arizona, Tim Green has worked as an analog and mixed-signal board/system level design engineer for over 23 years, including brushless motor control, aircraft jet engine control, missile systems, power op amps, data acquisition systems, and CCD cameras. Tim's recent experience includes analog & mixed-signal semiconductor strategic marketing. He is currently a Strategic Development Engineer at Burr-Brown, a division of Texas Instruments, in Tucson, AZ and focuses on instrumentation amplifiers and digitally-programmable analog conditioning ICs. He can be contacted at green_tim@ti.com

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