Operational Amplifier Stability
Part 3 of 15: R_O and R_{OUT}

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Part 3 of this series will focus on clarifying some common misconceptions regarding op amp "Output Resistance". We will define two important *different* output resistances for an op amp, R_O and R_{OUT}. R_O will become extremely useful when we start to stabilize op amp circuits which are driving capacitive loads. We will present easy techniques to derive R_O from op amp manufacturers' data sheets and in addition a couple of real world measurement techniques for those op amps whose data sheets do not contain a specification for R_O. We will also show a trick for using SPICE op amp models and R_O which will allow you to use the SPICE Loop Gain Test while including the effects of R_O (extremely useful for capacitive load drive circuits).

Definition and Derivation of R_O and R_{OUT}

R_O is here, and throughout this series, defined as the Open Loop Output Resistance of an op amp. R_{OUT} is defined as the Closed Loop Output Resistance of an op amp. Fig. 3.0 emphasizes the important difference between these two different resistances.

R_O = Op Amp ***Open Loop*** Output Resistance

R_{OUT} = Op Amp ***Closed Loop*** Output Resistance

Fig. 3.0: Definition of R_O and R_{OUT}

As hinted at in Fig. 3.0 slide R_O and R_{OUT} are related. R_{OUT} is R_O reduced by loop gain. Fig. 3.1 will define the op amp model used for the derivation of R_{OUT} from R_O. This simplified op amp model focuses solely on the basic DC characteristics of an op amp. A high input resistance (100mΩ to GΩ) is between –IN and +IN. R_{DIFF} develops an error voltage across it, V_E, due to the voltage differences between -IN and +IN. The error voltage, V_E, is amplified by the open loop gain factor Aol and becomes V_O. In series with V_O to the output, V_{OUT}, is R_O, the open loop output resistance.

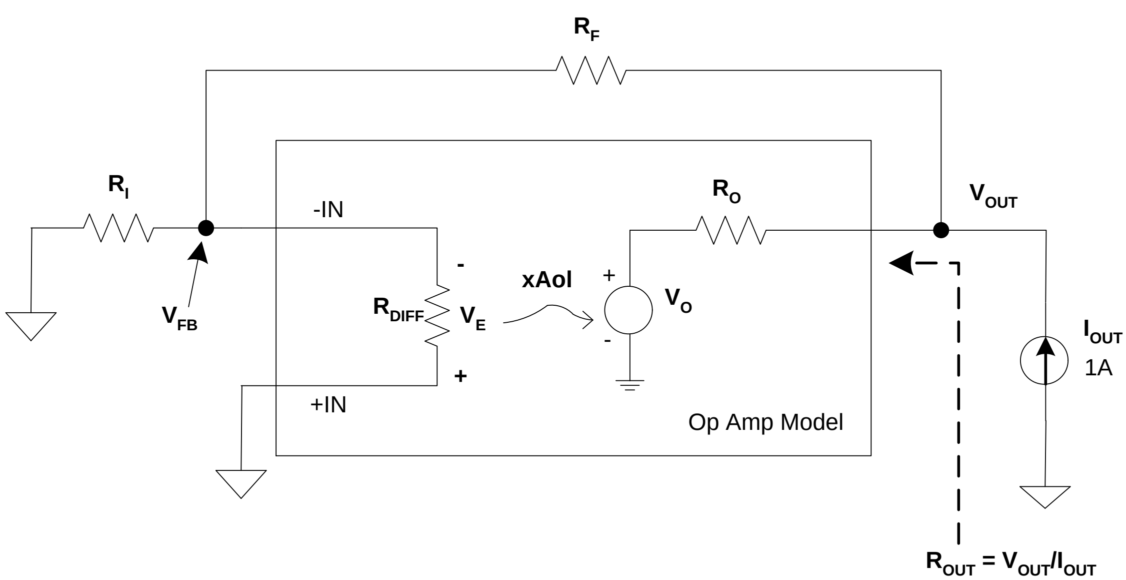


Fig. 3.1: Op Amp Model for Derivation of R_{OUT}

Using the op amp model in the Fig. 3.1 we can solve for R_{OUT} as a function of R_O and Aolβ. This derivation is detailed in Fig. 3.2. We see that Aolβ, loop gain, reduces R_O so that the output resistance of the op amp with feedback, R_{OUT}, will be much lower than R_O, for large values of Aolβ.

1) $b = V_{FB} / V_{OUT} = [V_{OUT} (R_I / \{R_F + R_I\})]/V_{OUT} = R_I / (R_F + R_I)$

2) $R_{OUT} = V_{OUT} / I_{OUT}$

3) $V_O = -V_E\, Aol$

4) $V_E = V_{OUT} [R_I / (R_F + R_I)]$

5) $V_{OUT} = V_O + I_{OUT}R_O$

6) $V_{OUT} = -V_E Aol + I_{OUT}R_O$ *Substitute 3) into 5) for* V_O

7) $V_{OUT} = -V_{OUT} [R_I/(R_F + R_I)]\, Aol + I_{OUT}R_O$ *Substitute 4) into 6) for* V_E

8) $V_{OUT} + V_{OUT} [R_I/(R_F + R_I)]\, Aol = I_{OUT}R_O$ *Rearrange 7) to get* V_{OUT} *terms on left*

9) $V_{OUT} = I_{OUT}R_O / \{1+[R_I Aol/(R_F+R_I)]\}$ *Divide in 8) to get* V_{OUT} *on left*

10) $R_{OUT} = V_{OUT}/I_{OUT} =[I_{OUT}R_O / \{1+[R_I Aol / (R_F+R_I)]\}] / I_{OUT}$

Divide both sides of 9) by I_{OUT} *to get* R_{OUT} *[from 2)] on left*

11) $R_{OUT} = R_O / (1+Aol\beta)$ *Substitute 1) into 10)*

$$R_{OUT} = R_O / (1+Aol\beta)$$

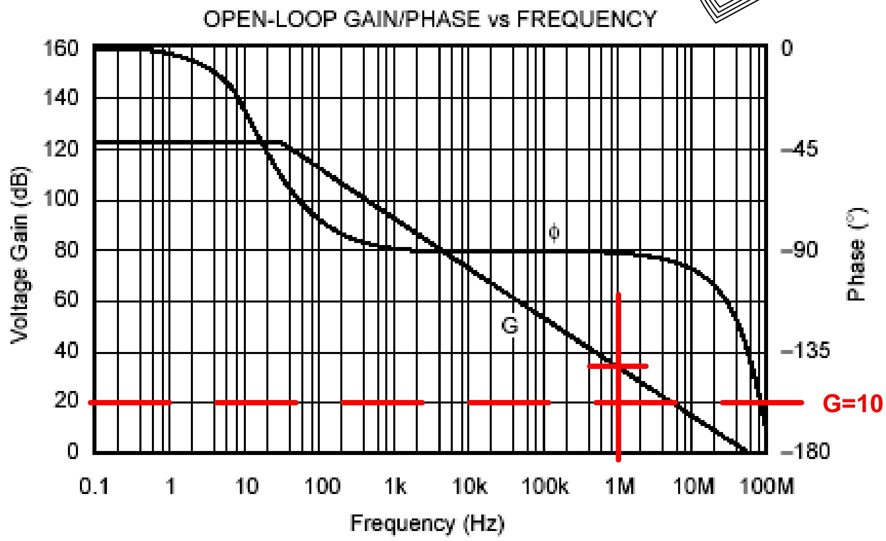
Fig. 3.2: Derivation of R_{OUT}

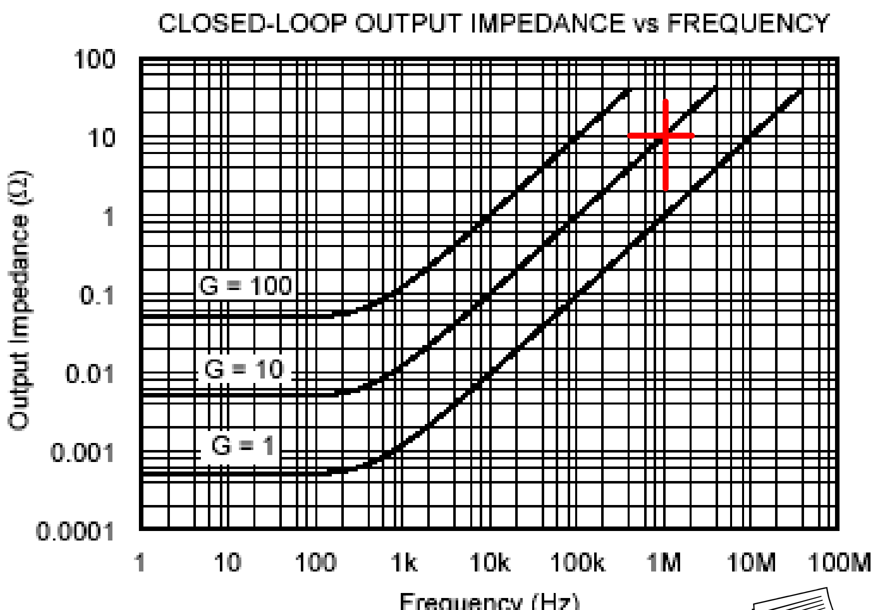
Computing R_O from Data Sheet Curves

The OPA353 is a wideband (UGBW=44MHz, SR=22V/uS, Settle to 0.1%=0.1us) CMOS, single supply (2.7V to 5.5V), RRIO (Rail-To-Rail Input and Output) op amp. There is no R_O specification in the Specifications table of the manufacturer's data sheet. However, there are two helpful curves in the Typical Performance Curves to help us determine R_O. We will need to use the Open-Loop Gain/Phase vs Frequency curve (see Fig. 3.3) and the Closed-Loop Output Impedance vs Frequency curve (see Fig. 3.4) to easily calculate R_O. The Closed-Loop Output Impedance vs Frequency curve is actually a plot of R_{OUT} vs frequency. Within the unity gain bandwidth of voltage feedback op amps R_O and R_{OUT} are predominantly resistive. On the Closed-Loop Output Impedance vs Frequency curve, Fig. 3.4, we choose the G=10 curve and on its x-axis the point 1MHz (just choose an easy to read data point). At the intersection of 1MHz and G=10 Curve we see R_{OUT}=10Ω. On the Open-Loop Gain/Phase vs Frequency curve, Fig. 3.3, we look at the 1MHz frequency point on the x-axis and read what the open loop gain as 29.54dB (We measured this one with a ruler and scaled it based on the linear dB y-axis. We did this on a cut and paste curve which was then enlarged as much as possible). The derivation of R_O from the information collected in Fig. 3.3 and Fig. 3.4 is detailed in Fig. 3.5. Now from our formula for R_O we rearrange the equation to give us R_O in terms of R_{OUT}, Aol, and β. From this equation and our data sheet information we calculate the R_O for the OPA353 to be 40Ω.

OPA353 Specifications:

Aol @1MHz = 29.54dB = x30





OPA353 Specifications:

R_{OUT} (@1MHz, G=10) = 10Ω

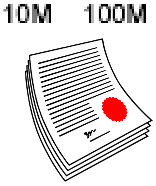


Fig. 3.4: OPA353 Closed Loop Output Impedance Curve

OPA353 R_O Calculation

$R_{OUT} = R_O / (1+Aol\beta)$

$R_O = R_{OUT} (1+Aol\beta)$

$R_O = 10\Omega (1+ 30[1/10])$

$R_O = 40\Omega$

Fig. 3.5: OPA353 R_O Calculation

We can use the op amp model from Fig. 3.1, used for the derivation of R_{OUT} from R_O, and the information from the OPA353 data sheet to fill in actual values in the model. This is shown in Fig 3.6. So we see how our model correlates with real world op amps. Notice in this model we define V_O as the op amp's output before R_O and V_{OUT} as the actual op amp output. Of course in a real op amp we can only gain access to V_{OUT} but this model and the fact that we can get real world data to build this model will become very powerful in stability analysis.

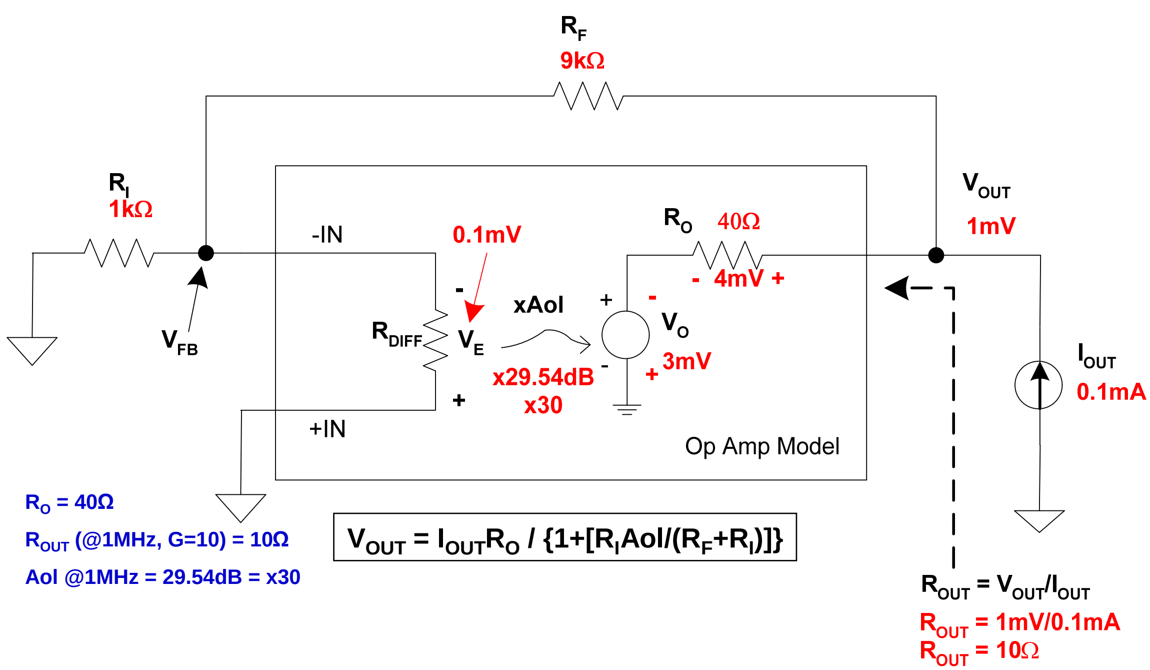


Fig. 3.6: OPA353 R_O Calculation using Op Amp Model

Summary of R_O and R_{OUT} Key Points

Fig 3.7 emphasizes the important differences between R_O and R_{OUT}. Fig 3.8 summarizes the key points of R_O.

- **R_O does *NOT* change when Closed Loop feedback is used**
- **R_{OUT} is the effect of R_O, Aol, and β controlling V_O**
  - **Closed Loop feedback (β) forces V_O to increase or decrease as needed to accommodate V_O loading**
  - **Closed Loop (β) increase or decrease in V_O appears at V_{OUT} as a reduction in R_O**
  - **R_{OUT} increases as Loop Gain ($Aol\beta$) decreases**

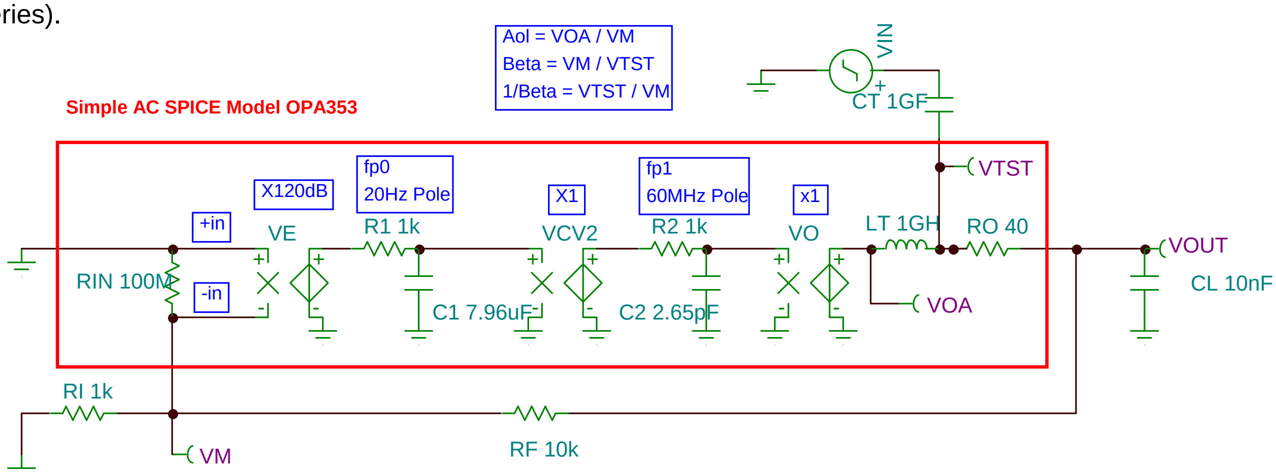
Fig. 3.7: R_O vs R_{OUT}

- Ø **R_O *is constant over the Op Amp's bandwidth*** 
- Ø ***R_O is defined as the Op Amp's Open Loop Output Resistance***
- Ø ***R_O is measured at I_{OUT} = 0 Amps, f = 1MHz***
  (use the unloaded R_O for Loop Stability calculations since it will be the largest value à worst case for Loop Stability analysis)
- Ø ***R_O is included when calculating b for Loop Stability analysis***

Fig. 3.8: R_O Key Points

R_O and SPICE Simulations

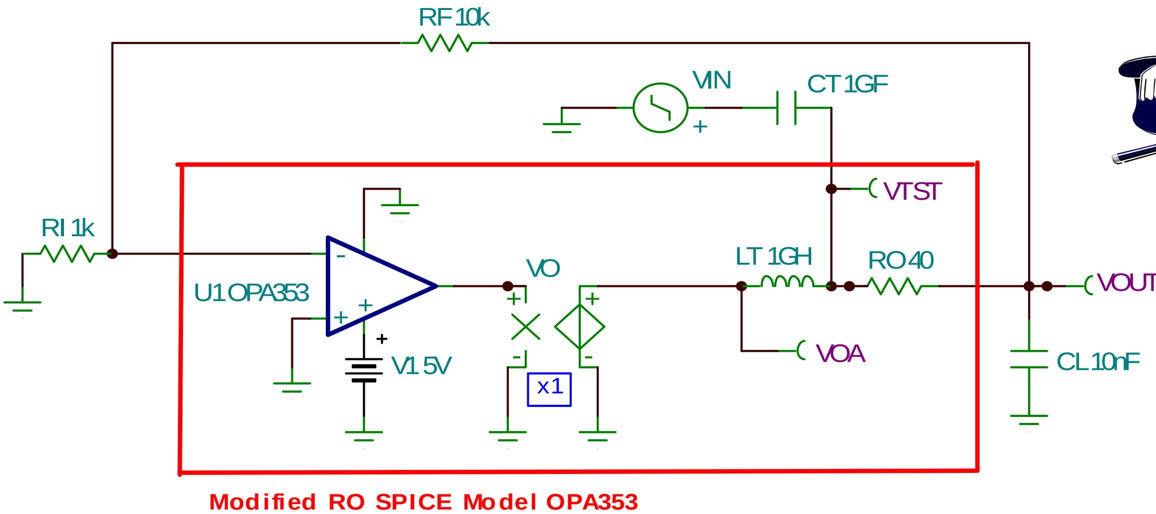
In Fig. 3.9 we show a Simple AC SPICE Model for the OPA353. Here we use the 40Ω we computed for R_O. Notice that we break the loop for AC Stability Analysis here using the SPICE Loop Gain Test. The loop break is made between RO and VO in order to analyze the effects of R_O on 1/β. This will become extremely important in stabilizing capacitive loads driven by op amps (this topic will be covered in detail in Part 7 and Part 8 of this series).

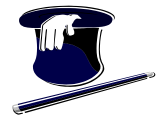


SPICE Loop Gain Test - Break the loop between VO and RO

Fig. 3.9: Simple AC SPICE Model with R_O

For a given existing manufacturer's op amp SPICE model we can easily add an external R_O so that when we use the SPICE Loop Gain Test to find 1/β we can include the effects of R_O. In the Modified R_O SPICE MacroModel in Fig. 3.10 we add a Voltage Controlled Voltage Source (VCVS), VO, with a gain equal to one. This isolates the op amp's output and any internal R_O it may have modeled internally from whatever connects to VOA. Now we can add our own RO after the VCVS, VO, and break the loop between VO and RO which is desired for including the effects of R_O when analyzing capacitive loads and their effects on 1/β.





U1 is Mfr SPICE Model

Add VO (VCVS w/G=1) and new RO

Allows SPICE Loop Gain Test 1/β curve to include effects of RO

Fig. 3.10: Modified R_O SPICE MacroModel

Real World R_O for Single Supply Op Amps

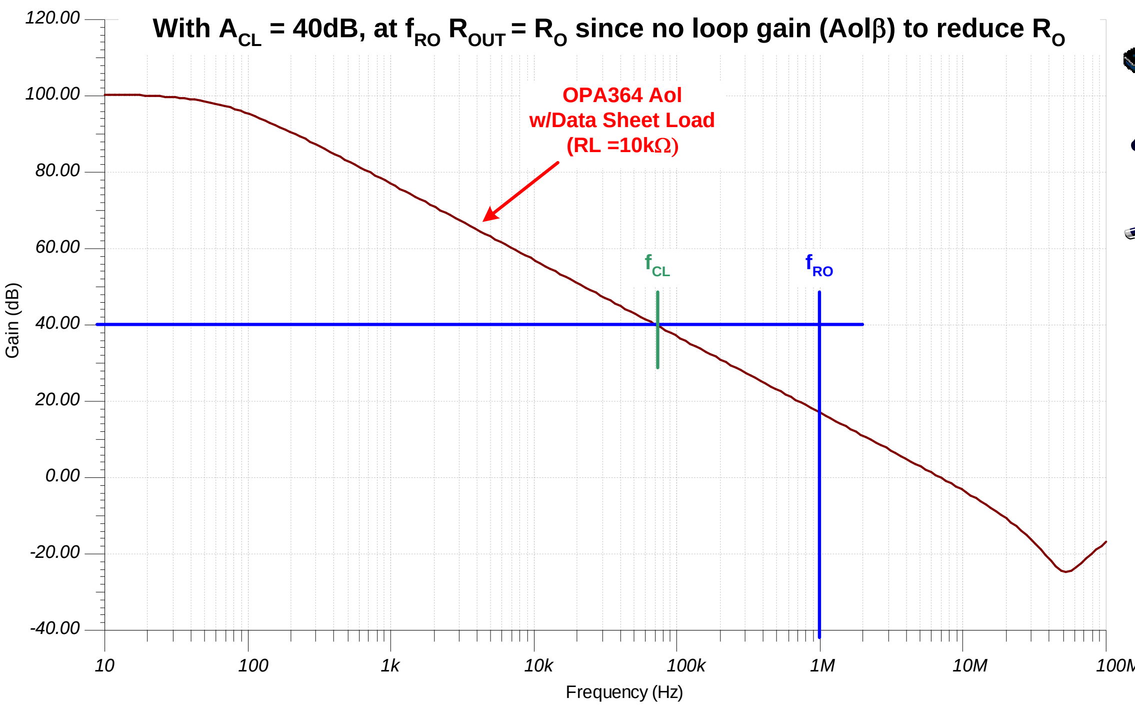
Fig.3.11 lists some actual real world measured R_O for many single supply op amps. Notice that the OPA353 we analyzed to be R_O=40Ω has a measured value of 44Ω. This close correlation is because the data we used from the manufacturer's data sheet was also measured data on a typical part!

Part	R_O (ohms)	Part	Ro (ohms)	Part	Ro (ohms)
OPA132	80	OPA348	600	OPA627	55
OPA227	40	OPA350	50	OPA684	50
OPA277	10	OPA353	44	THS4503	14
OPA300	20	OPA354	35	TLC080	100
OPA335	90	OPA355	40	TLC081	100
OPA336	250	OPA356	30	TLC2272	140
OPA340	80	OPA363	160	TLE2071	80
OPA343	80	OPA380	30	TLV2461	173

Fig. 3.11: Real World R_O for Some Single Supply Op Amps

Real World Measurement Techniques for R_O

So what if we do not have any manufacturer's specifications for R_O and we want to know what it is? There are two real world techniques we can use to measure R_O. Each method starts by looking at the Open-Loop Gain/Phase vs Frequency curve. Such a curve is shown in Fig. 3.12 for the OPA364, a wideband (UGBW=7MHz, SR=5V/uS, Settle to 0.1%=1.5us) CMOS, single supply (1.8V to 5.5V), RRIO (Rail-To-Rail Input and Output) op amp with "Linear Offset Over Common Mode Range". If we choose to test this op amp at a gain of 100 and at 1MHz there will be no loop gain, Aolβ, left. Therefore, if we measure R_{OUT} under these conditions we will really be getting a value for R_O!



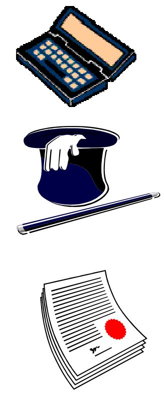


Fig. 3.12: Trick for Measuring R_O

The test circuit in Fig. 3.13 shows one method for measuring R_O in the real world. This method we will call the R_O – Drive Method. Here the output of the OPA364 is driven through an AC coupling capacitor, C1. This is to ensure we do not load down the amplifier with any DC current. Most op amps' R_O gets smaller as large currents are driven through them. We want to measure R_O at its highest value (which will cause the most problem during AC stability analysis). In this technique the voltage at the output of the amplifier, VO, is measured. Also measured is the voltage, VTest, at the junction of the AC coupling capacitor, C1, and the current limiting resistor, R3. The current into the op amp's output is calculated and used to divide the voltage at the op amp to give us the measured R_O value! Note that although the OPA364 is a single supply op amp (1.8V to 5.5V) we can cleverly run it at +2.5V and -2.5V to avoid a more complicated level-shifting of our input or output signal.

NOTE: *All measurements used in the "Drive Method" must be AC voltages without any DC component. If one uses the AC Analysis / Calculate Nodal Voltages in TINA SPICE he will get an rms voltage reading at the nodes which includes the DC voltages in the circuit (i.e. offset referred-to-output). If the Offset Voltage is significant in comparison to the AC Voltage components then an erroneous R_O will be calculated! In Fig. 3.13 the AC Analysis / Calculate Nodal Voltages was used but the DC Offset at VOA is about 87.63µV in comparison to 34.87mV and 353.55mV rms values which are dominated by AC voltage components.*

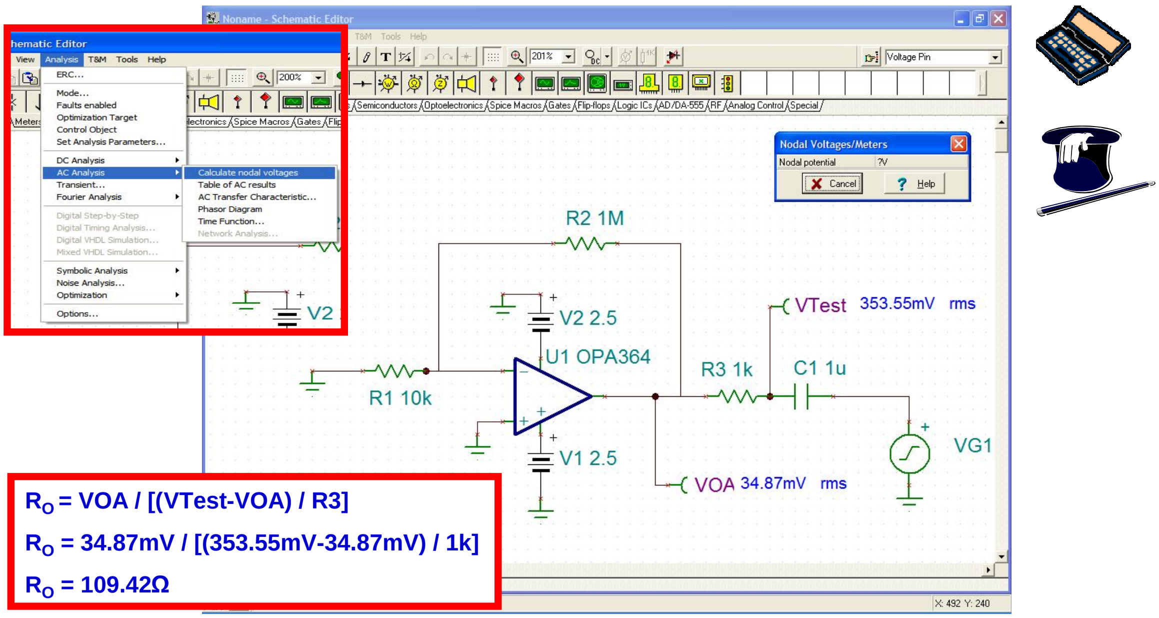


Fig. 3.13: Measuring R_O – Drive Method

The test circuits in Fig. 3.14 and Fig. 3.15 show another method for measuring R_O in the real world. This technique will take a voltage reading out of the op amp both loaded and unloaded and then compute the value for R_O. We will still need to use a high gain and high frequency combination to ensure there is no loop gain reducing R_{OUT} for our measurements. In this configuration a small AC signal is injected into the op amp's input. Inverting or non-inverting gain will work. In Fig. 3.14 we measure VOUT, the unloaded voltage. Note that it is a small value of output so that when we do load it we will not pull much current since we are looking for the unloaded (and therefore highest value) of R_O.

NOTE: *All measurements used in the "Load Method" must be AC voltages without any DC component. If one uses the AC Analysis / Calculate Nodal Voltages in TINA SPICE he will get an rms voltage reading at the nodes which includes the DC voltages in the circuit (i.e. offset referred-to-output). If the Offset Voltage is significant in comparison to the AC Voltage components then an erroneous R_O will be calculated!*

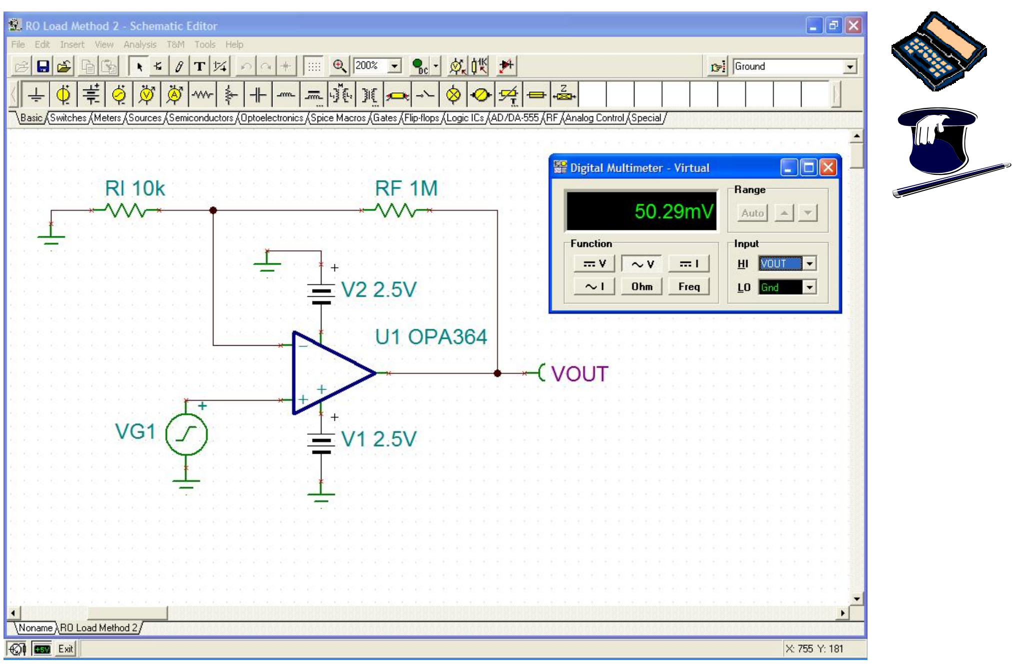
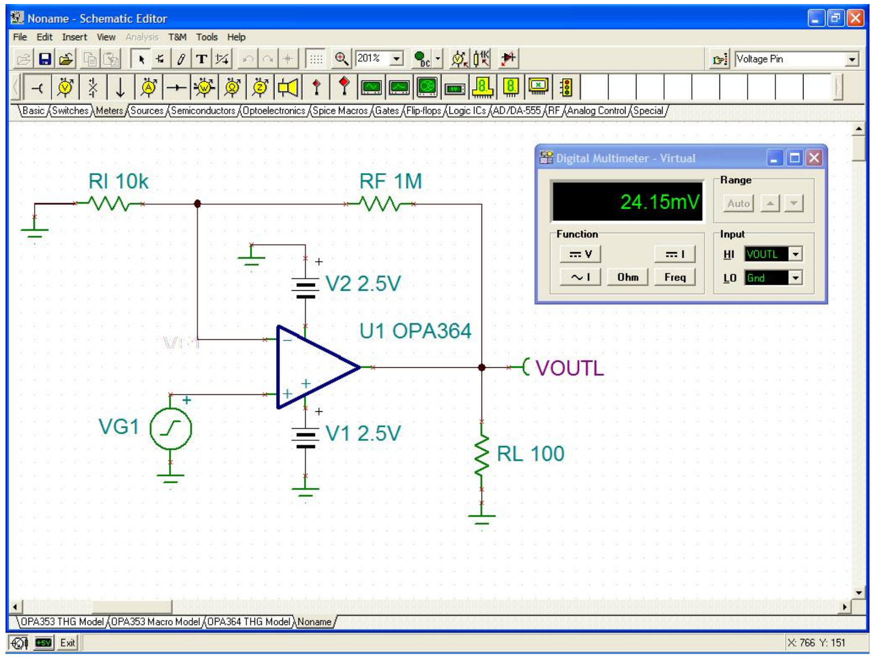


Fig. 3.14: Measuring R_O – Load Method, V_{OUT} Unloaded

In Fig. 3.15 we measure VOUTL, the loaded value of VOUT when RL is attached to the output of the op amp. Note how the value of RL does not cause large currents to flow into or out of the op amp's output.



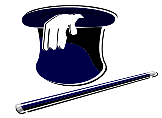
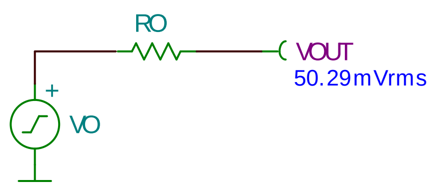
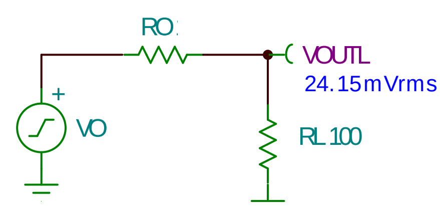
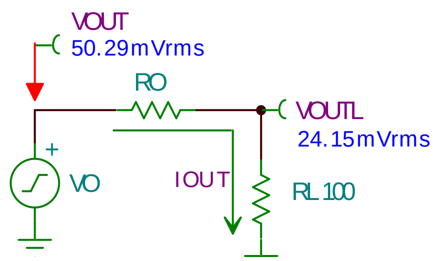


Fig. 3.15: Measuring R_O – Load Method, V_{OUT} Loaded

Now we have completed our measurements for the R_O-Load Method a simple calculation will result in the value for R_O. The unloaded value, VOUT, is always there at VO whether or not a load, RL, is present. From this we can create the final model shown in Fig. 3.16. IOUT, is, by inspection just VOUTL / RL. The drop across RO is VOUT-VOUTL. The voltage drop across RO divided by the current through it will give us the value for RO as shown in this slide. Notice that this method yields $R_O = 108.2\Omega$ and the R_O-Drive Method yielded $R_O = 109.42\Omega$. Either method is acceptable to measure real world R_O.







OPA364 R_O Calculation:

1) IOUT = VOUTL / RL

IOUT = 24.15mV / 100

IOUT = 0.2415mA

2) RO = (VOUT – VOUTL) / IOUT

RO = (50.29mV – 24.15mV) / 0.2415mA

R_O = 108.2Ω

3) RO = [RL (VOUT – VOUTL)] / VOUTL

{Substitute 1) into 2) and solve for RO}

Fig. 3.16: Measuring R_O – Load Method Calculation

References:

Frederiksen, Thomas M. *Intuitive Operational Amplifiers*, *From Basics to Useful Applications*, Revised Edition. McGraw-Hill Book Company. New York, New York. 1988