Stability for Op Amps
Part 1 of 15: Loop Stability Basics

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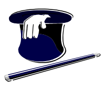
1.0 Introduction

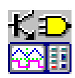
The overall technique used for this entire series will be "definition by example" wherever possible with generic formulae included for use in other applications. To make stability analysis easy we will be using more than one tool in our analysis toolbox. Data Sheet Information, Tricks, Rules-Of-Thumb, SPICE Simulation, and Real-World Testing will all be used to accelerate our design of stable Op Amp circuits. These tools are specifically targeted for Voltage Feedback Op Amps with Unity Gain Bandwidths <20MHz although many of the techniques are applicable to any voltage feedback op amp. The reason <20MHz is chosen is that as we increase to wider bandwidth Op Amp circuits there are other major factors in closing the loop such as parasitic capacitances on PCBs, parasitic inductances in capacitors, parasitic inductances and capacitances in resistors, etc. Most of the rules of thumb and techniques were developed not only in theory but from practical design and building of real world circuits with Op Amps <20MHz.

Part 1 of this series review some basic fundamentals essential to ease of stability analysis and define some nomenclature which will be used consistently throughout the entire series.

ü Data Sheet Info 

ü Tricks 

ü Rules-Of-Thumb 

ü Tina SPICE Simulation 

ü Testing 

Goal: To learn how to *EASILY* analyze and design Op Amp circuits for guaranteed Loop Stability using Data Sheet Info, Tricks, Rules-Of-Thumb, Tina SPICE Simulation, and Testing.

Note: Tricks & Rules-Of-Thumb apply for Voltage Feedback Op Amps, Unity Gain Bandwidth <20MHz

Figure 1.0 Stability Analysis Toolbox

1.1 Bode Plot Basics

The frequency response for the Magnitude Plot is the change in voltage gain as frequency changes. This change is specified on a Bode plot, a plot of voltage gain in dB (decibels) versus frequency (Hz). Bode Magnitude Plots are plotted as semi-log plots with frequency (Hz) on the x-axis, log scale, and voltage gain (dB) on the y-axis, linear scale. Preferred y-axis scaling is a convenient 20dB per major division. The other half of the Bode Plot is the Phase Plot (phase shift versus frequency) and is plotted as degrees phase shift versus frequency. Bode Phase Plots are plotted as semi-log plots with frequency (Hz) on the x-axis, log scale, and phase shift (degrees) on the y-axis, linear scale. Preferred y-axis scaling is a convenient 45 degrees per major division.

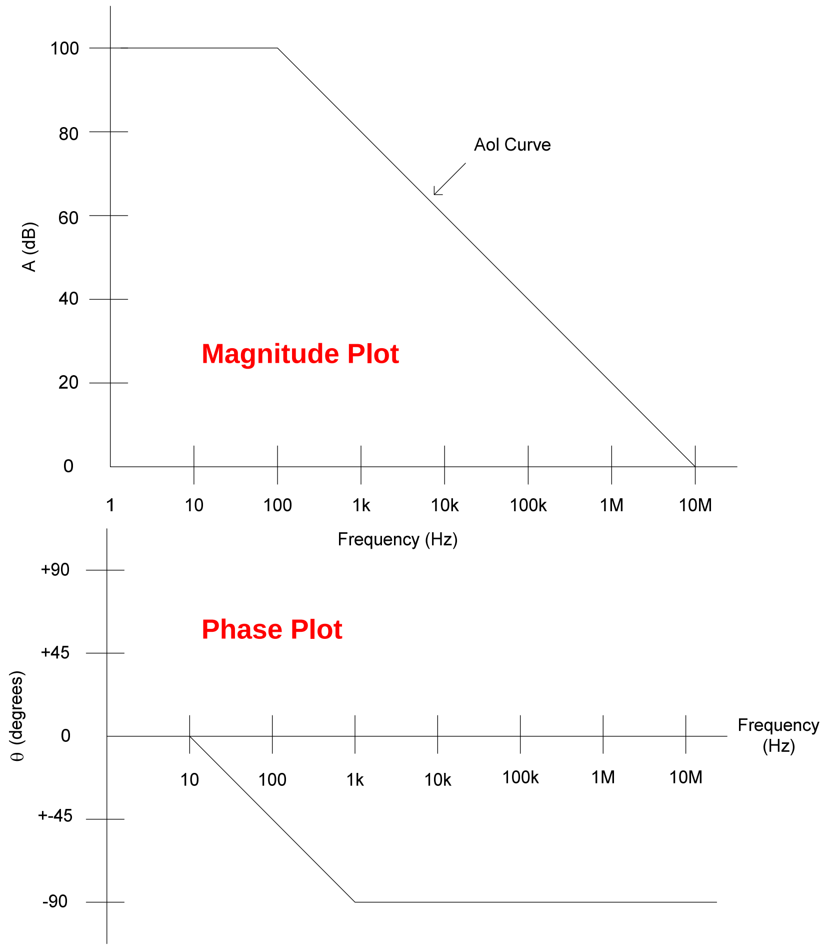


Figure 1.1 Magnitude and Phase Bode Plots

Magnitude Bode Plots require voltage gain to be converted to dB (Decibel). For our gain analysis we will use Voltage Gain with dB defined as $20Log_{10}A$, where A is defined as the voltage gain in Volts/Volts.

dB à A(dB) = $20Log_{10}A$ where A = Voltage Gain in V/V

A (V/V)	**A (dB)**
0.001	**-60**
0.01	**-40**
0.1	**-20**
1	**0**
10	**20**
100	**40**
1,000	**60**
10,000	**80**
100,000	**100**
1,000,000	**120**
10,000,000	**140**

Figure 1.2 dB (decibel) Definition for Magnitude Bode Plots

Figure 1.3 defines some commonly used Bode Plot terms.

- Roll-Off Rate à Decrease in gain with frequency
- Decade à x10 increase or x1/10 decrease in frequency. From 10Hz to 100Hz is one decade.
- Octave à X2 increase or x1/2 decrease in frequency. From 10Hz to 20Hz is one octave.

Figure 1.3 More Bode Plot Definitions

On a voltage gain Bode Plot the slope of voltage gain with frequency is defined in +20dB/decade or -20dB/decade increments. Another way of describing the same slopes is +6dB/octave or -6dB/octave (see Figure 1.4)

The following derivation proves the equivalent nature of 20dB/decade and 6dB/octave.

$\Delta A(dB) = A(dB)$ at fb – $A(dB)$ at fa
$\Delta A(dB) = [Aol(dB) - 20\log10(fb/f1)] – [Aol(dB) - 20\log10(fa/f1)]$
$\Delta A(dB) = Aol(dB) - 20\log10(fb/f1) – Aol(dB) + 20\log10(fa/f1)]$
$\Delta A(dB) = 20\log10(fa/f1) – 20\mathrm{Log}10(fb/f1)]$
$\Delta A(dB) = 20\log10(fa/fb)$
$\Delta A(dB) = 20\log10(1k/10k) = -20dB/decade$

$\Delta A(dB) = 20\log10(fb/fc)$
$\Delta A(dB) = 20\log10(10k/20k) = -6db/octave$

-20dB/decade = -6dB/octave

Therefore:
+20dB/decade = +6dB/octave   -20dB/decade = -6dB/octave
+40dB/decade = +12dB/octave   -40dB/decade = -12dB/octave
+60dB/decade = +18dB/Octave   -60dB/decade = -18dB/Octave

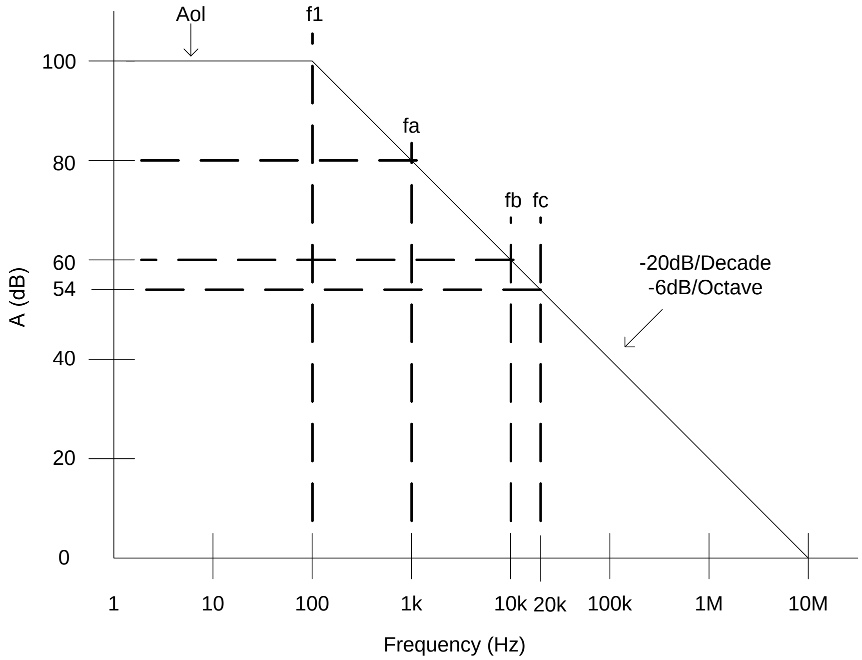
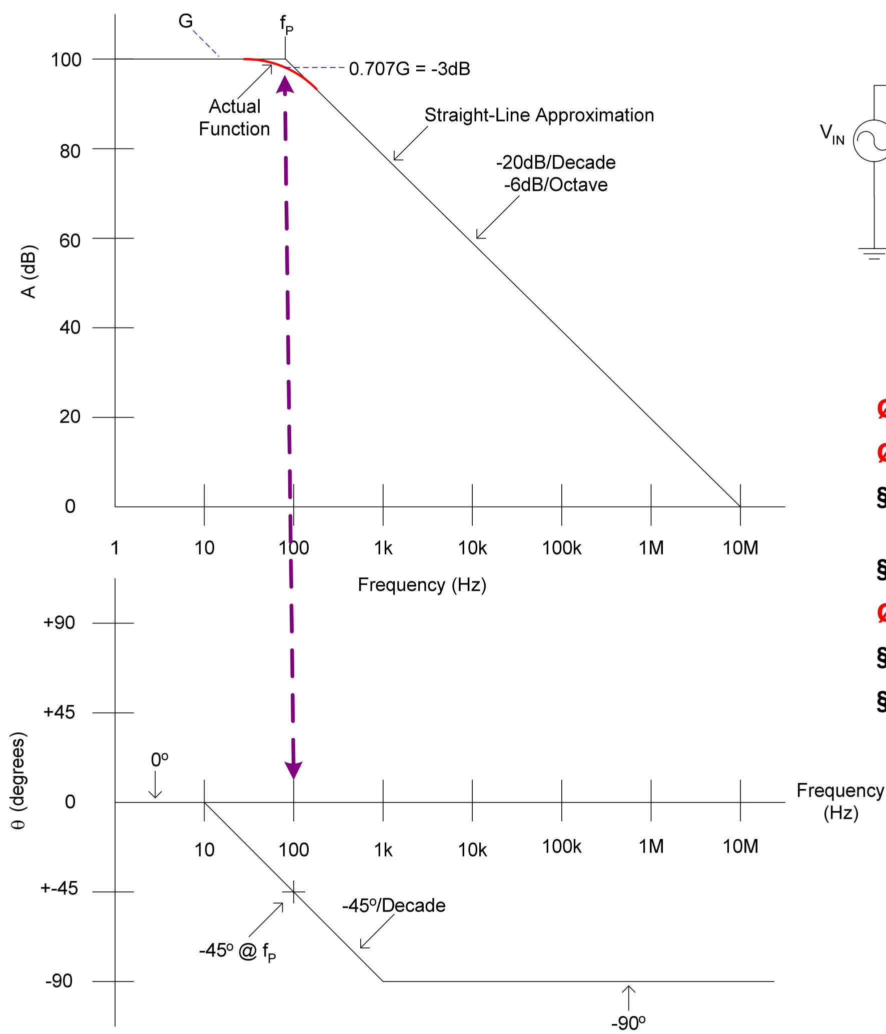
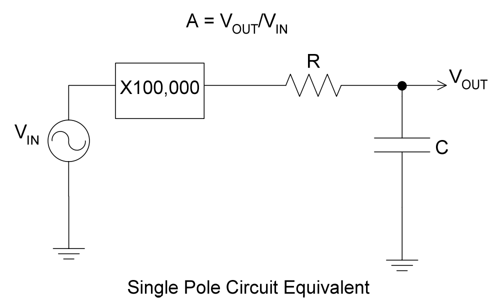


Figure 1.4 Magnitude Bode Plot: 20dB/Decade = 6dB/Octave

Pole à A single pole response has a -20dB/decade, -6db/octave roll-off in the Bode plot (magnitude or gain plot). At the pole location the gain is reduced by -3dB from the DC value of the gain. In the phase plot the pole has a -45° phase shift at f_P, the frequency of the pole location. The phase extends on either side of f_P to 0° and -90° at a -45°/decade slope. A single pole may be represented by a simple RC low pass network as shown in Figure 1.5. Notice how the phase of a pole affects frequencies up to one decade above and one decade below the pole location in frequency.





Ø **Pole Location** = f_P

Ø **Magnitude** = -20dB/Decade Slope

§ Slope begins at f_P and continues down as frequency increases

§ Actual Function = -3dB down @ f_P

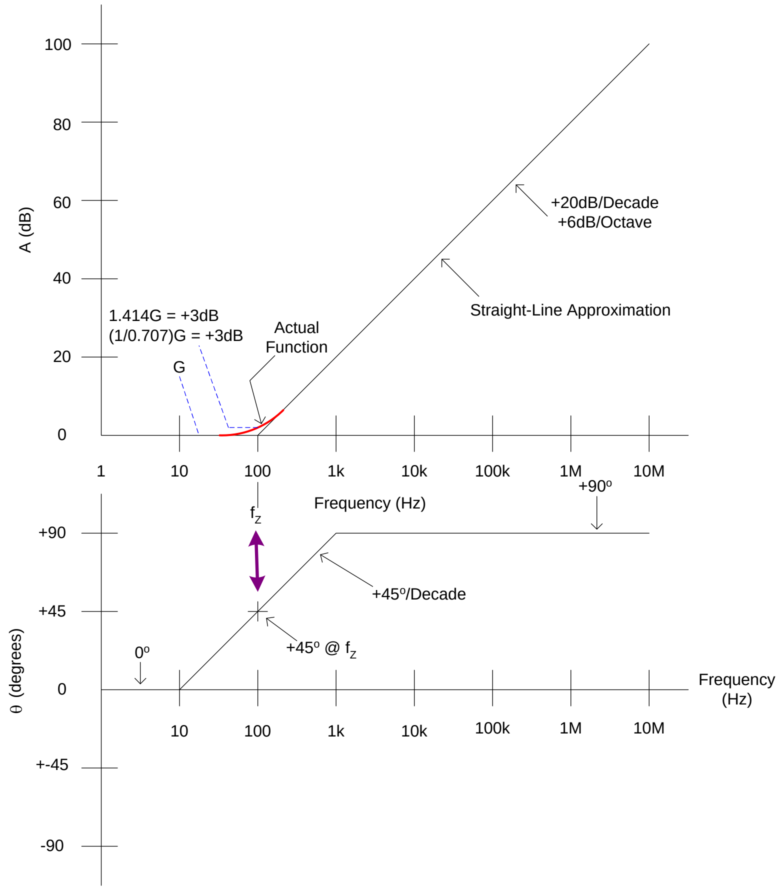
Ø **Phase** = -45°/Decade Slope through f_P

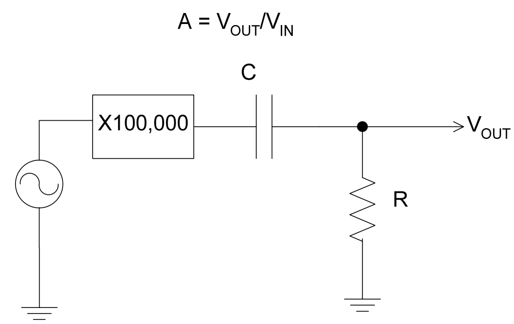
§ Decade Above f_P Phase = -90°

§ Decade Below f_P Phase = 0°

Figure 1.5 Poles: Bode Plot Magnitude and Phase

Zero à A single zero response has a +20dB/decade, +6db/octave "roll-on" (opposite of roll-off?) in the Bode plot (magnitude or gain plot). At the zero location the gain is increased by +3dB from the DC value of the gain. In the phase plot the pole has a +45° phase shift at f_Z, the frequency of the zero location. The phase extends on either side of f_Z to 0° and +90° at a +45°/decade slope. A single zero may be represented by a simple RC high pass network as shown in Figure 1.6. Notice how the phase of a zero affects frequencies up to one decade above and one decade below the zero location in frequency.





Single Zero Circuit Equivalent

Ø **Zero Location** = f_Z

Ø **Magnitude** = +20dB/Decade Slope

§ Slope begins at f_Z and continues up as frequency increases

§ Actual Function = +3dB up @ f_Z

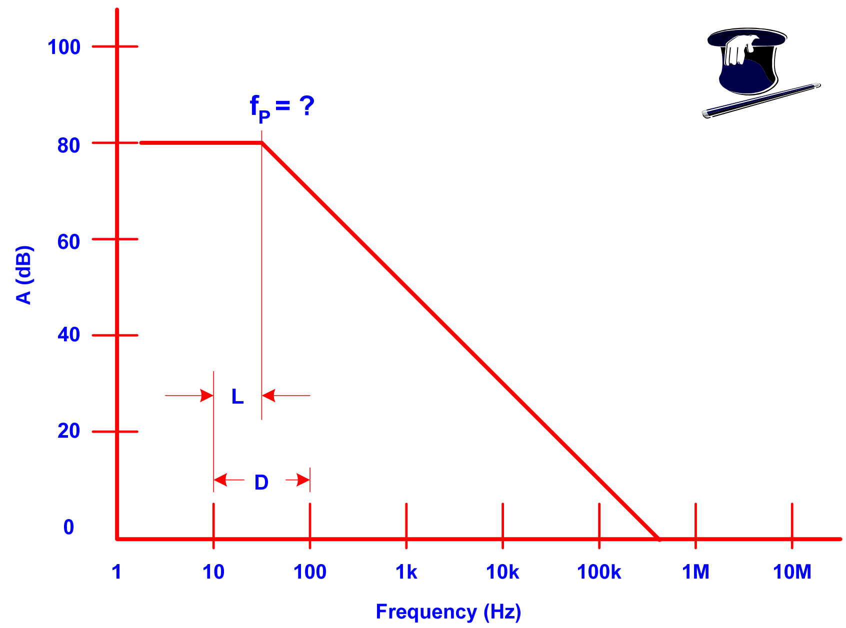
Ø **Phase** = +45°/Decade Slope through f_Z

§ Decade Above f_Z Phase = +90°

§ Decade Below f_Z Phase = 0°

Figure 1.6 Zeros: Bode Plot Magnitude and Phase

On a Bode Magnitude Plot it is easy to measure the frequency location of a given pole or zero. Since the x-axis is a log scale of frequency this technique allows a ratio of distances to accurately and quickly determine the frequency of the pole or zero of interest. Figure 1.7 illustrates this "Log Scale Trick".



Log Scale Trick (f_P = ?):

1) Given: L = 1cm; D = 2cm

2) $L/D = \text{Log}_{10}(f_P)$

3) $\mathbf{f_P = Log_{10}^{-1}(L/D) = 10^{(L/D)}}$
   $\mathbf{f_P = 10^{(L/D)} = 10^{(1cm/2cm)} = 3.16}$

4) Adjust for the decade range working within –
   10Hz-100Hz decade à
   f_P = 31.6Hz

5) $\mathbf{L = Log_{10}(fp') \times D}$
   $\mathbf{L = Log_{10}(3.16) \times 2cm = 1cm}$
   where fp' = fp normalized to the 1-10 decade range –
   f_P = 31.6 à f_P' = 3.16

Figure 1.7 Log Scale Trick

1.2 Intuitive Component Models

Most op amp applications use combinations of four key components: op amp, resistor, capacitor, and inductor. To facilitate stability analysis it is convenient to have "intuitive models" for these key components.

Our Intuitive Op Amp Model for AC Stability Analysis is defined in Figure 1.8. The differential voltage between the IN+ and IN- terminals will be amplified by x1 and converted to a single-ended AC Voltage Source, V_{DIFF}. V_{DIFF} is then amplified by K(f). K(f) represents the data sheet Aol Curve (Open Loop Gain vs Frequency Plot). This resultant voltage, V_O, is then followed by the Op Amp Open Loop, AC Small Signal, Output Resistance, R_O. After passing through R_O the voltage appears as V_{OUT}.

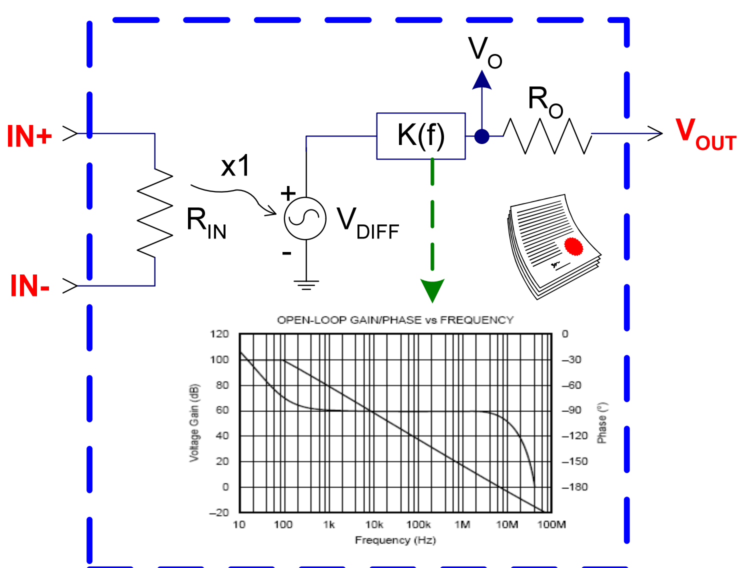


Figure 1.8 Intuitive Op Amp Model

Our Intuitive Resistor Model for AC Stability Analysis is defined in Figure 1.9. The resistor has a constant impedance value regardless of the frequency it is operating at.

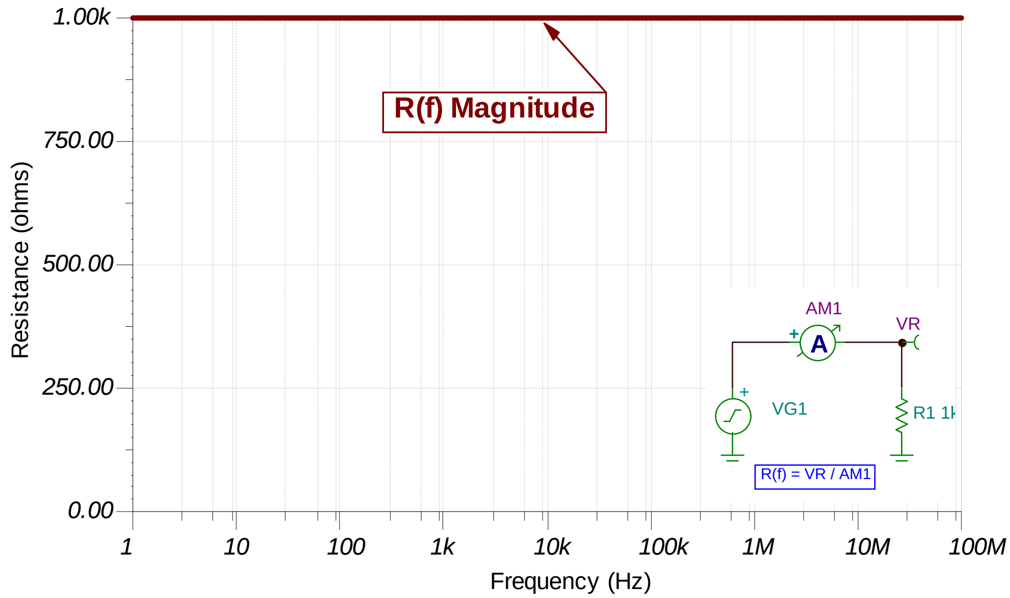


Figure 1.9 Intuitive Resistor Model

Our Intuitive Capacitor Model for AC Stability Analysis is defined in Figure 1.10 and contains three distinct operating areas. At "DC" the capacitor will be viewed as an open circuit. At "High Frequency" the capacitor will be viewed as a short circuit. In between the capacitor will be viewed as a frequency controlled resistor with a 1/Xc decrease in impedance as frequency increases. The SPICE simulation result in Figure 1.11 depicts our Intuitive Capacitor Model over frequency.

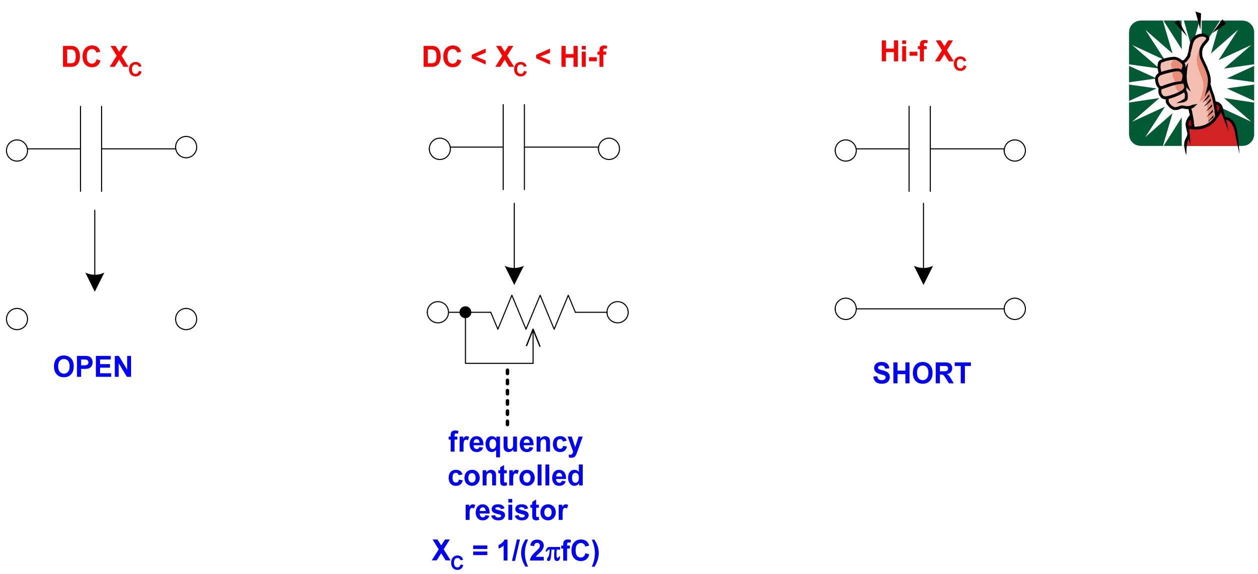


Figure 1.10 Intuitive Capacitor Model

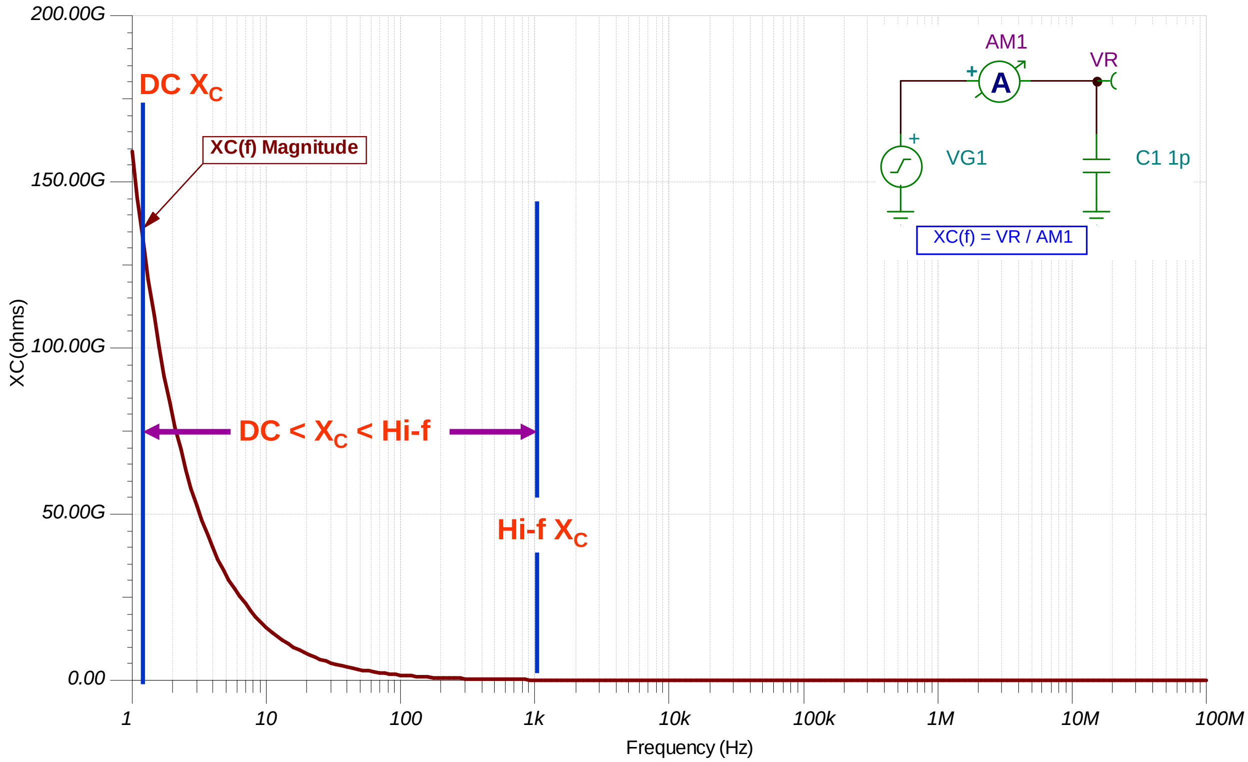


Figure 1.11 Intuitive Capacitor Model SPICE Simulation

Our Intuitive Inductor Model for AC Stability Analysis is defined in Figure 1.12 and contains three distinct operating areas. At "DC" the inductor will be viewed as a short circuit. At "High Frequency" the inductor will be viewed as an open circuit. In between the capacitor will be viewed as a frequency controlled resistor with an X_L increase in impedance as frequency increases. The SPICE simulation result in Figure 1.13 depicts our Intuitive Inductive Model over frequency.

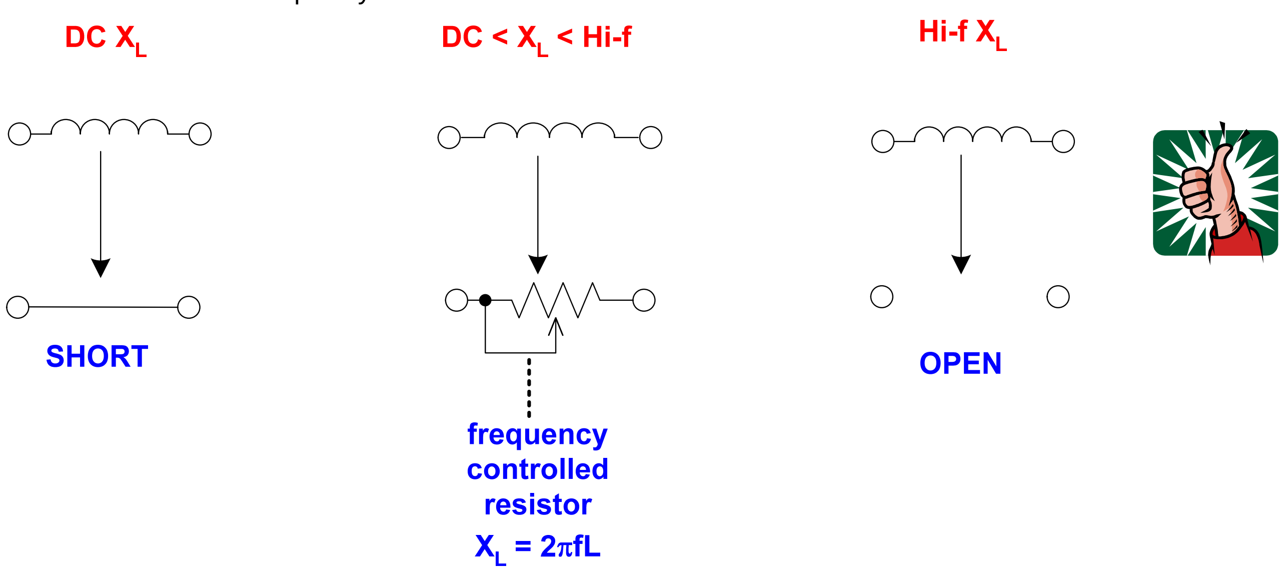


Figure 1.12 Intuitive Inductor Model

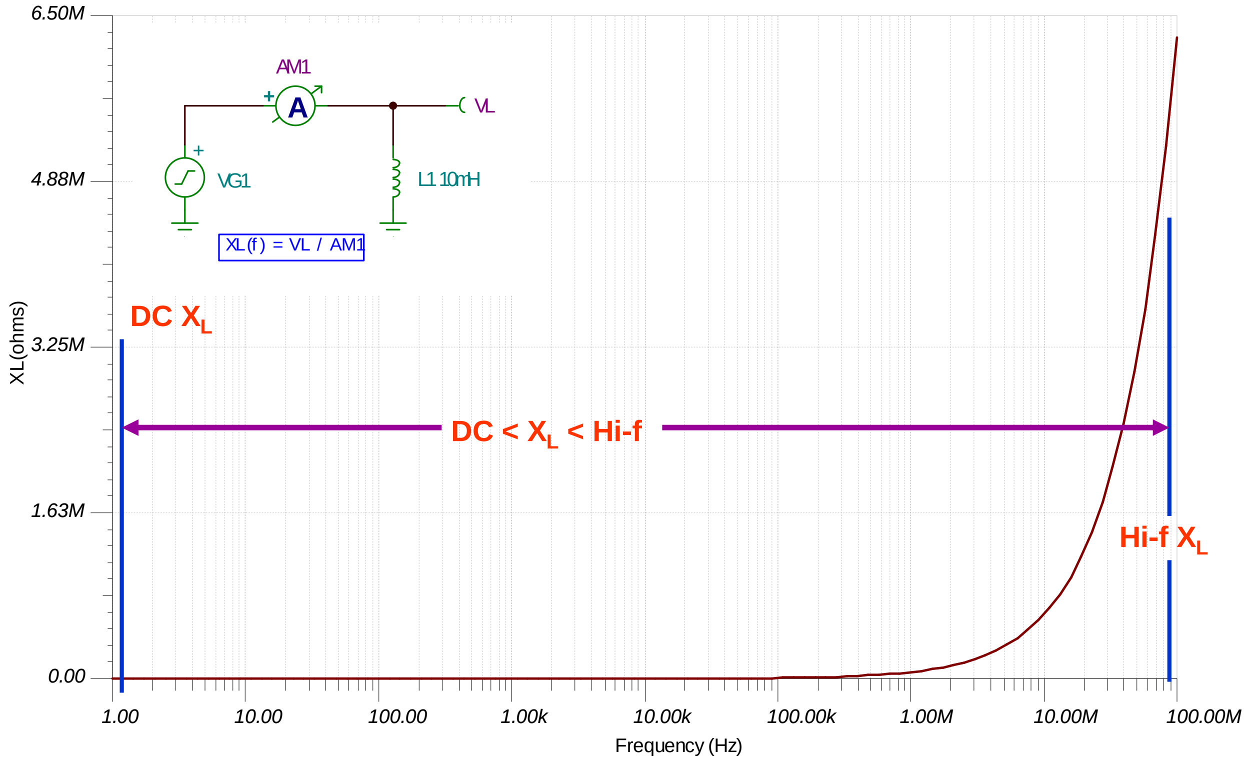
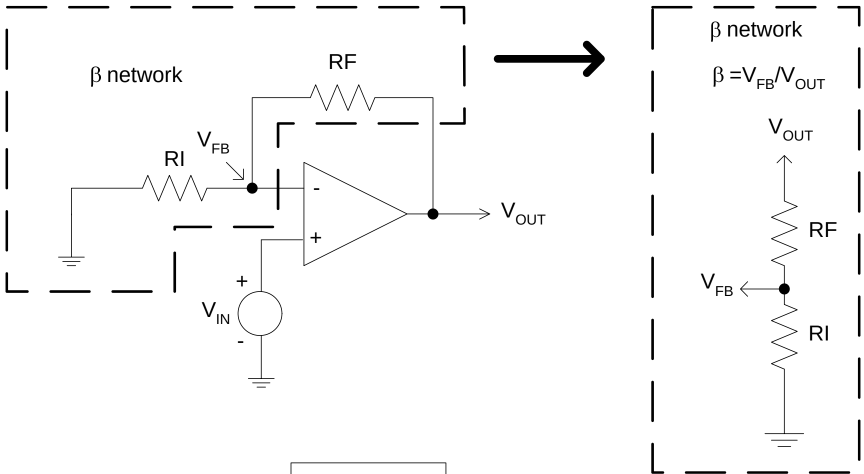


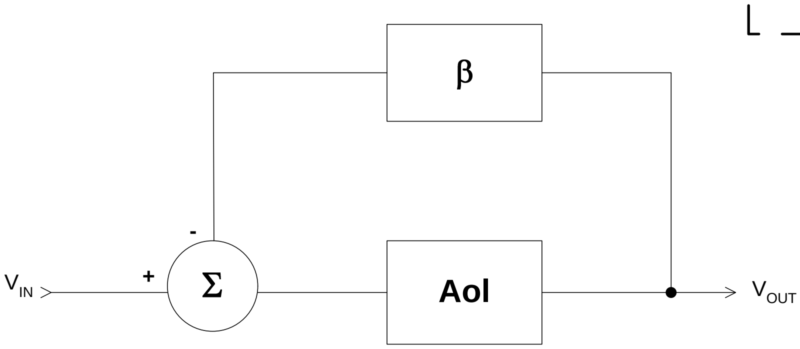
Figure 1.13 Intuitive Inductor Model SPICE Simulation

1.3 Stability Criteria

The lower part of Figure 1.14 illustrates the traditional control loop model block diagram which represents an op amp circuit with feedback. The top part of Figure 1.14 depicts the sections of a typical op amp circuit with feedback which correspond to the control loop model. This model of an op amp circuit with feedback we will call the Op Amp Loop Gain Model. Notice that the Aol is the Op Amp data sheet parameter Aol, and is the open loop gain of the op amp. β (Beta) is the amount of output voltage from V_{OUT} which gets fed back as feedback. The β network in this example is a resistor feedback network.

In the derivation of V_{OUT}/V_{IN} we see that the closed loop gain function is directly defined by Aol and β.





V_{OUT}/V_{IN} = Acl = Aol/(1+Aolβ)

If Aol >> 1 then Acl ≈ 1/β

Aol: Open Loop Gain

β: Feedback Factor

Acl: Closed Loop Gain

Figure 1.14 Op Amp Loop Gain Model

From our Op Amp Loop Gain Model in Figure 1.14, we can derive the criteria for a stable closed loop op amp circuit. This is detailed in Figure 1.15.

At the frequency fcl, where Loop Gain (Aolβ) goes to 1 or 0dB, if the Loop Gain Phase Shift is +/-180° then we have instability! At fcl the distance the Loop Gain Phase Shift is from 180° is called the Loop Gain Phase Margin. Our desired Loop Gain Phase Margin is >45° for a critically damped well-behaved closed loop response.

V_{OUT}/V_{IN} = Aol / (1+ Aolβ)
If: Aolβ = -1
Then: V_{OUT}/V_{IN} = Aol / 0 à ∞

If V_{OUT}/V_{IN} = ∞ à Unbounded Gain
Any small changes in V_{IN} will result in large changes in V_{OUT} which will feed back to V_{IN} and result in even larger changes in V_{OUT} à *OSCILLATIONS* à *INSTABILITY* !!

Aolβ: Loop Gain
Aolβ = -1 à Phase shift of ±180°, Magnitude of 1 (0dB)
fcl: frequency where Aolβ = 1 (0dB)

Stability Criteria:
At fcl, where Aolβ = 1 (0dB), Phase Shift < ±180°
Desired Phase Margin (distance from ±180° Phase Shift) ≥ 45°

Figure 1.15 Derivation of Stability Criteria

1.4 Loop Stability Tests

Since Loop Stability is defined by the magnitude and phase plot of Loop Gain (Aolβ) then we will need to know how to easily analyze Loop Gain magnitude and phase. To do this we will break the closed loop op amp circuit and inject a small signal AC source into the loop and measure amplitude and phase to plot the complete loop gain picture. Figure 1.16 shows the equivalent control loop block diagrams for the Op Amp Loop Gain Model and the technique we will use for the Loop Gain Test.

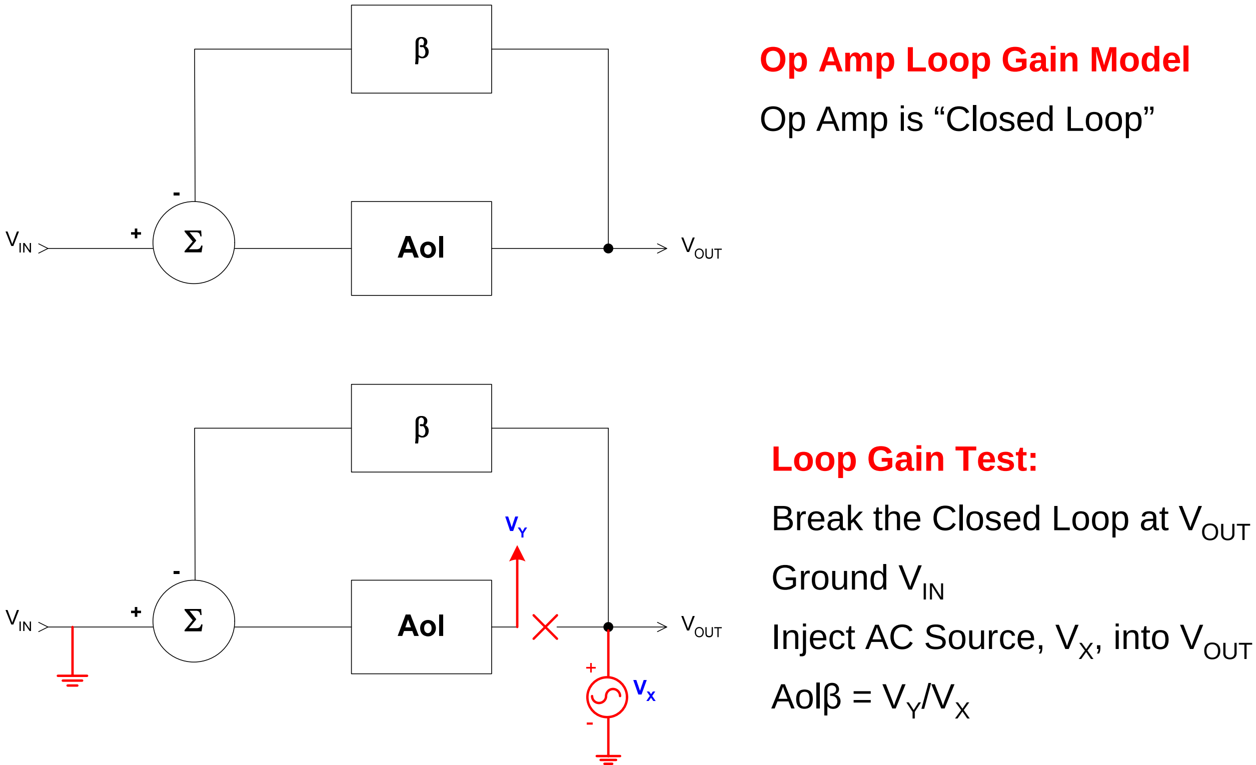
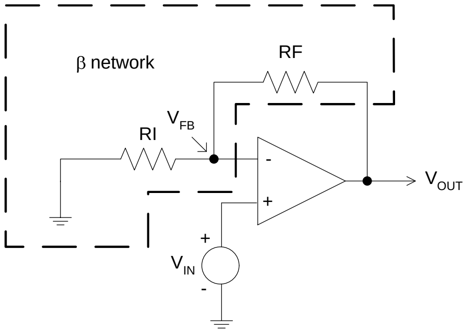


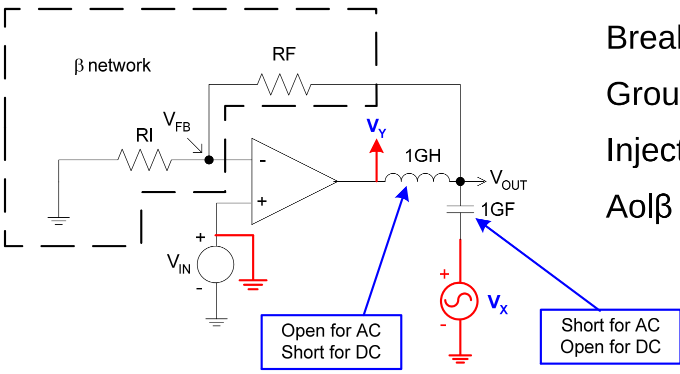
Figure 1.16 Traditional Loop Gain Test

When analyzing a circuit built in SPICE for simulation the Traditional Loop Gain Test breaks the closed loop op amp circuit through the use of an inductor and capacitor. A very large value of inductance ensures the loop is closed at DC (a requirement for SPICE simulation is to be able to calculate a DC Operating Point first before performing an AC Analysis) but open at AC frequencies of interest. A very large value of capacitance ensures that our AC Small Signal Source is not connected at DC but is directly connected at the frequencies of interest. Figure 1.17 illustrates the SPICE setup schematic for the Traditional Loop Gain Test.



Op Amp Loop Gain Model

Op Amp is "Closed Loop"



SPICE Loop Gain Test:

Break the Closed Loop at V_{OUT}

Ground V_{IN}

Inject AC Source, V_X, into V_{OUT}

$Aol\beta = V_Y/V_X$

Figure 1.17 Traditional Loop Gain Test – SPICE Setup

Before we simulate a circuit in SPICE we will want to know what the approximate outcome will be. Remember GIGO (Garbage In Garbage Out)!! Beta (β) and its reciprocal 1/Beta (1/β) along with the data sheet Aol Curve will provide a powerful method for first-order approximations of Loop Gain analysis before we run SPICE. In future sections tricks and rules-of-thumb will be presented for computing Beta (β) and its reciprocal (1/β). Figure 1.18 defines the Beta (β) network for Op Amp circuits.

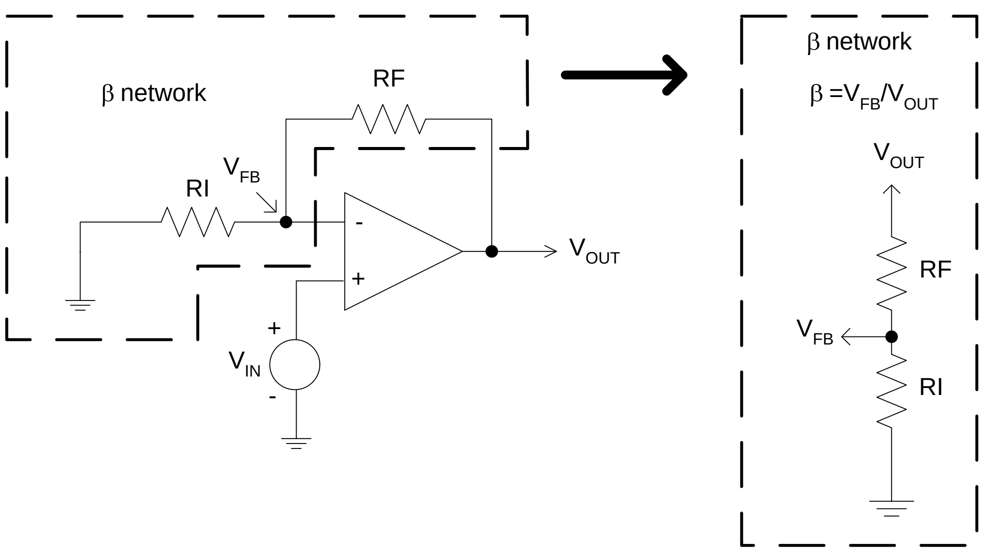


Figure 1.18 Op Amp β Network

The 1/β plot imposed on the Aol curve will provide a clear picture of exactly what the Loop Gain (Aolβ) plot is. From the derivation in Figure 1.19 we clearly see that the Aolβ magnitude plot is simply the difference between Aol and 1/β when we plot 1/β in dB on an Aol curve. Note that as frequency increases Aolβ decreases. Aolβ is the gain left to correct for errors in the V_{OUT}/V_{IN} or Closed Loop response. So as Aolβ decreases the V_{OUT}/V_{IN} response will become less accurate until the point where Aolβ goes to 0dB where from then on the V_{OUT}/V_{IN} response will simply follow the Aol curve.

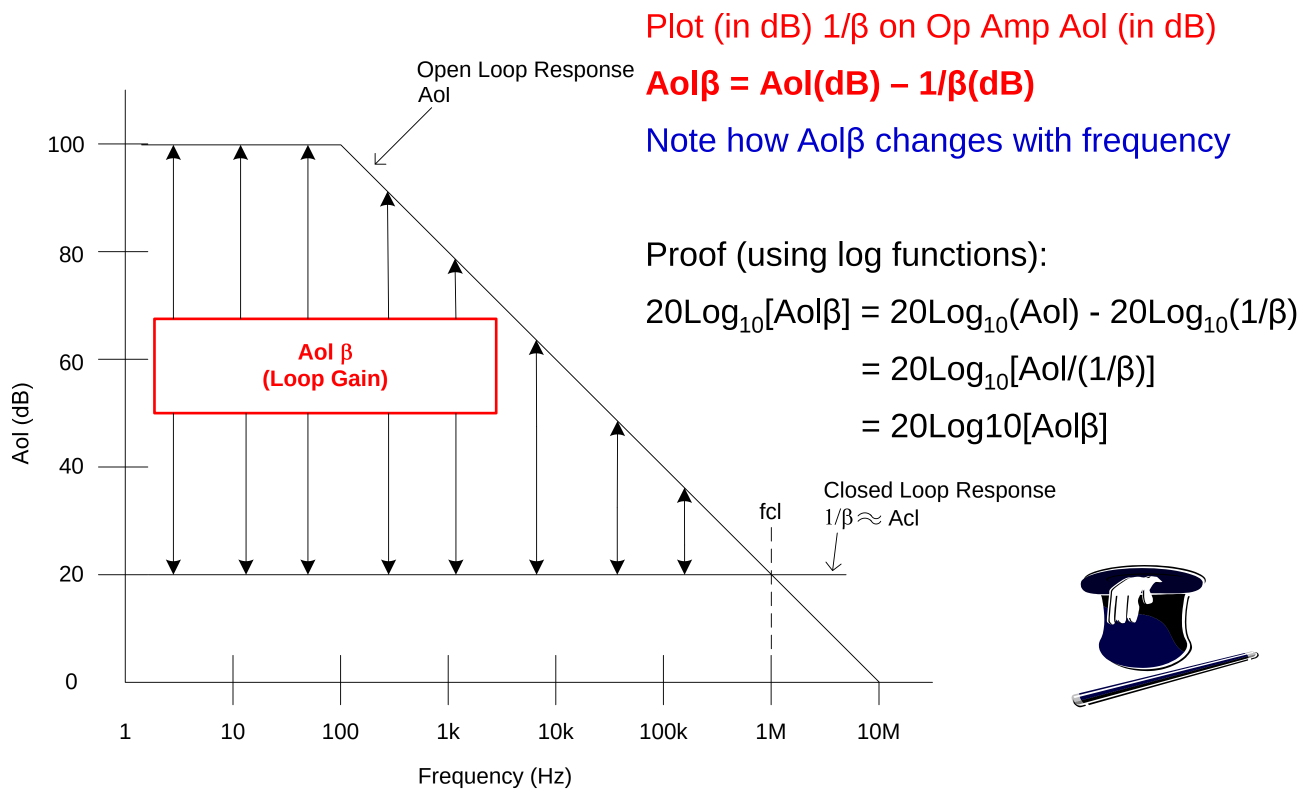


Figure 1.19 Loop Gain Information from Aol Plot and 1/β Plot

Once we plot 1/β on the Aol curve there is an easy first-order check for stability called Rate-Of-Closure. This Rate-Of-Closure stability check is defined as the "rate of closure" of 1/β curve with the Aol curve at fcl, where loop gain goes to 0dB. A 40db/decade rate-of-closure implies an UNSTABLE circuit and a 20dB/decade rate-of-closure implies a STABLE circuit. The 40dB/decade rate-of-closure implies instability because it implies two poles in the Aolβ plot before fcl which can mean a 180 phase shift. Four examples are shown in Figure 1.20 with their respective rate-of-closure computed below.

fcl1: Aol-1/β1 = -20dB/decade - +20dB/decade = -40dB/decade ◊ 40dB/decade rate-of-closure & UNSTABLE
fcl2: Aol-1/β2 = -20dB/decade - 0dB/decade = -20dB/decade ◊ 20dB/decade rate-of-closure & STABLE
fcl3: Aol-1/β3 = -40dB/decade - 0dB/decade = -40dB/decade ◊ 40dB/decade rate-of-closure & UNSTABLE
fcl4: Aol-1/β4 = -40dB/decade - -20dB/decade = -20dB/decade ◊ 20dB/decade rate-of-closure & STABLE

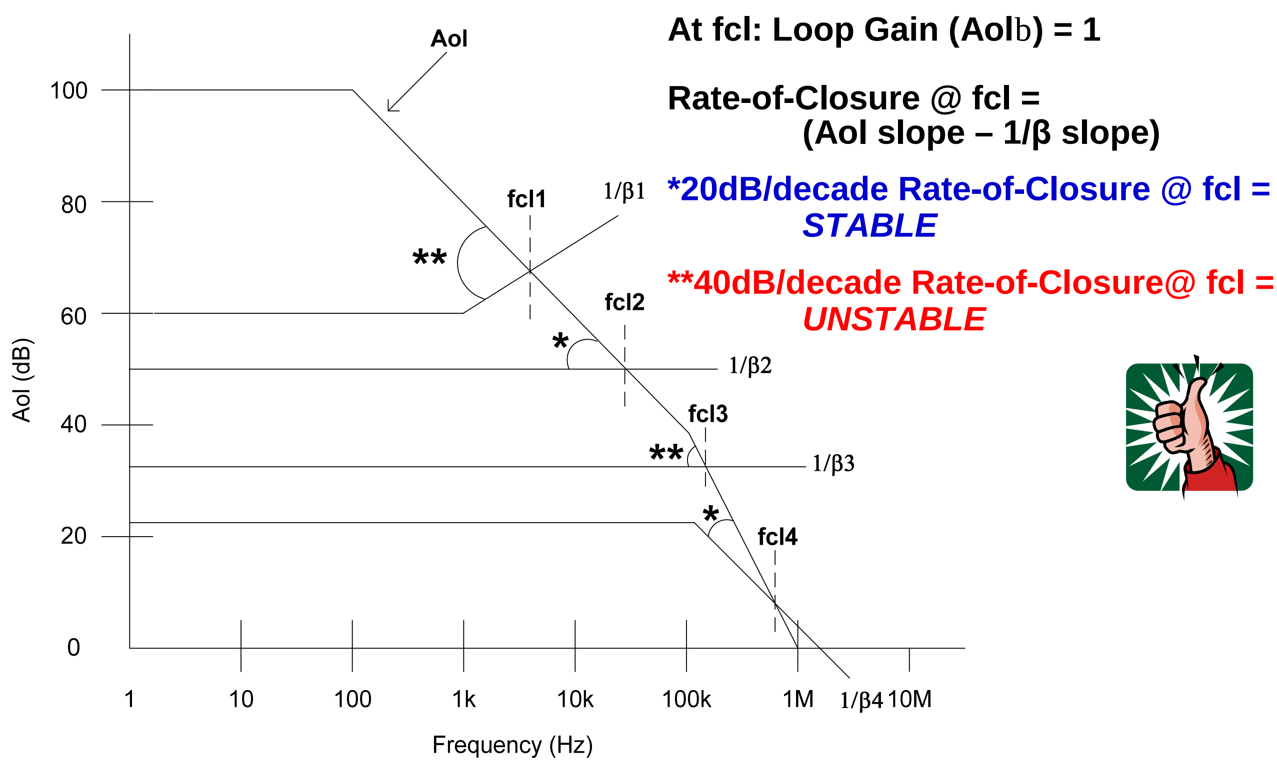


Figure 1.20 Rate-of-Closure Test for Loop Gain Stability

1.5 Loop Gain Stability Example

A loop gain analysis example (see Figure 1.21) serves to relate how we can analyze the stability of an op amp circuit from the 1/β plot plotted on the Aol curve. Here as frequency increases the capacitor CF goes towards a short in impedance thereby lowering the magnitude of the β plot with frequency (less voltage feedback as frequency increases) and respectively raising the 1/β curve as frequency increases. From our rate-of-closure criteria we predict an UNSTABLE circuit.

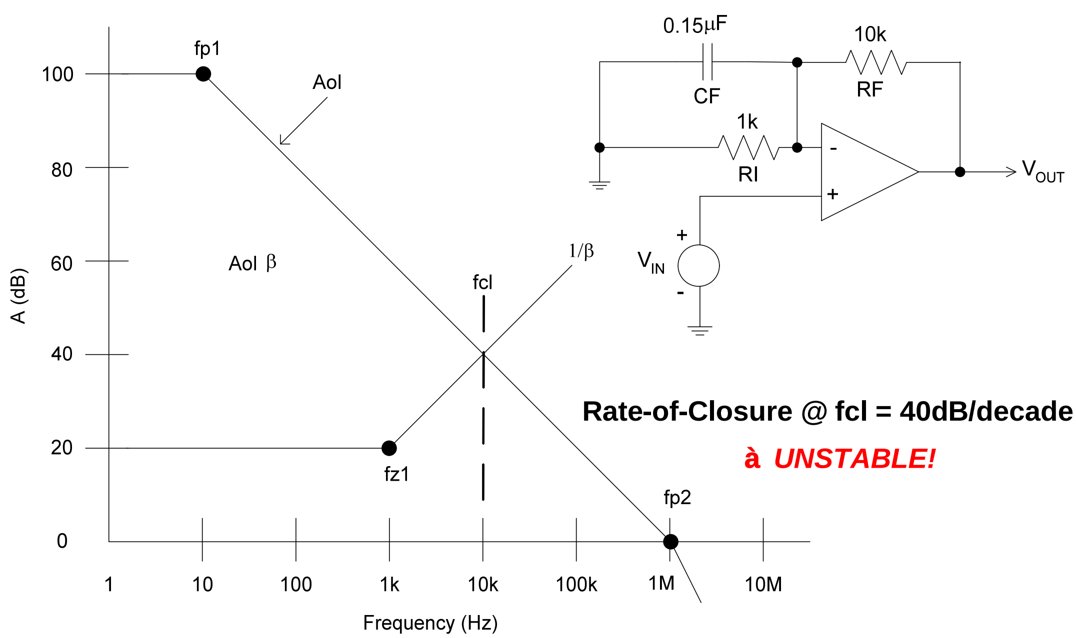
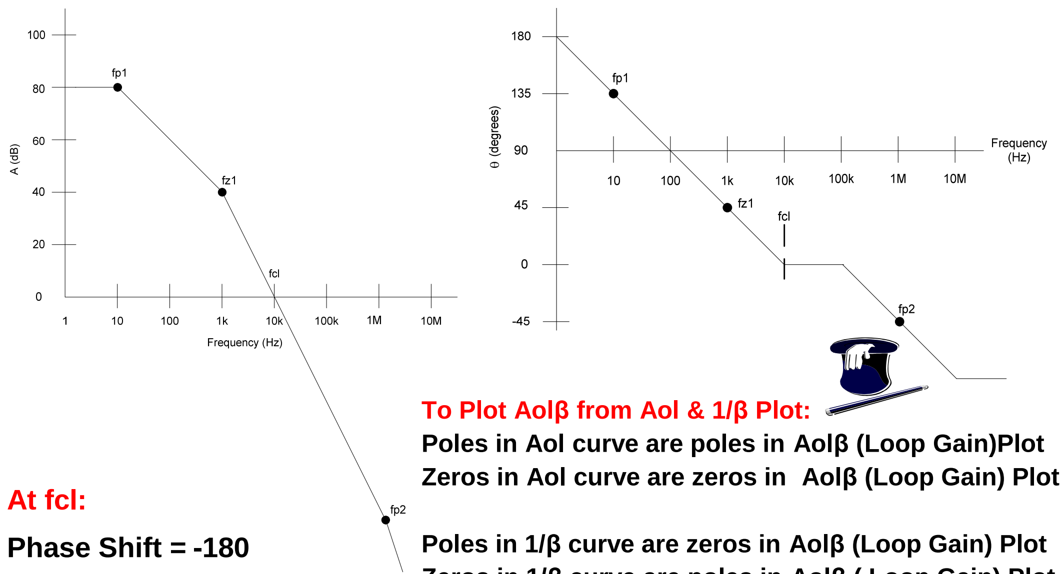


Figure 1.21 Loop Gain Stability Example

From our 1/β plot on the Aol curve we can plot the Aolβ (Loop Gain) magnitude plot (see Figure 1.22). From the Loop Gain magnitude plot we can then plot the Loop Gain phase plot. The rules to create an Aolβ plot from the 1/β plot on the Aol curve are simple: Poles and zeros from the Aol curve are poles and zeros in the AolB plot. Poles and zeros from the 1/β plot are opposite in the Aolβ plot. One easy way to remember this is β is used in the Aolβ plot and 1/β is the reciprocal of β and so we would expect the Aolβ curve to use the reciprocal of poles and zeros from the 1/β plot. Reciprocal of a pole is a zero and reciprocal of a zero is a pole.



To Plot Aolβ from Aol & 1/β Plot:
Poles in Aol curve are poles in Aolβ (Loop Gain)Plot
Zeros in Aol curve are zeros in Aolβ (Loop Gain) Plot

Poles in 1/β curve are zeros in Aolβ (Loop Gain) Plot
Zeros in 1/β curve are poles in Aolβ (Loop Gain) Plot
[Remember: β is the reciprocal of 1/β]

At fcl:

Phase Shift = -180

Phase Margin = 0

Figure 1.22 Loop Gain Plot from Aol Curve & 1/β Plot

1.6 1/β and Closed Loop Response

The V_{OUT}/V_{IN} closed loop response is not always the same as 1/β. In the example in Figure 1.23 we see that the AC small signal feedback is modified by the Rn-Cn network in parallel with RI. As frequency increases we see the results of this network reflected in the 1/β plot on the Aol curve. Think of this example as an inverting summing op amp circuit. We are summing in V_{IN} through RI and Ground through the Rn-Cn netowrk. V_{OUT}/V_{IN} will not be affected by this Rn-Cn network at low frequencies and the desired gain is seen as 20dB. As Loop Gain (Aolβ) is forced to 1 (0dB) by the Rn-Cn network there is no loop gain left to correct for errors and V_{OUT}/V_{IN} will follow the Aol curve at frequencies above fcl.

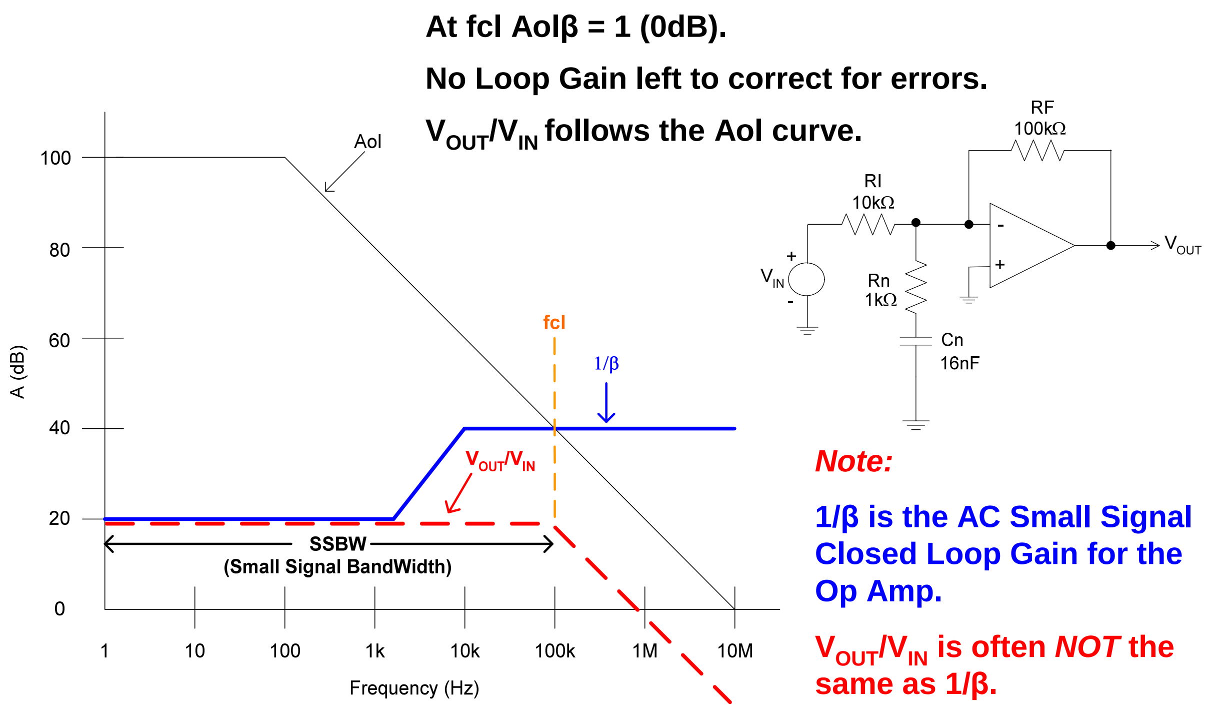


Figure 1.23 V_{OUT}/V_{IN} vs 1/β

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