Analysis and Measurement of Intrinsic Noise in Op Amp Circuits Part X: Instrumentation Amplifier Noise

by Art Kay, Senior Applications Engineer, Texas Instruments Incorporated

This part of our TechNote focuses on noise analysis and simulation in instrumentation amplifier circuits. We also discuss methods for minimizing noise in instrumentation amplifier designs.

Short Review of Three-Amp Instrumentation Amplifier

<u>Instrumentation amplifiers (INAs)</u> amplify small differential signals. Most INAs contain several resistors and operational amplifiers (op amps). While it is possible to build them using discrete components, there are many advantages to using monolithic integrated circuit INAs, including accuracy and size.

Fig. 10.1 shows the topology of a three-amplifier INA as well as some of the key connections. This is the most popular topology for instrumentation amplifiers. In this section, we will develop the gain equation for the INA: an important equation for noise analysis. This TechNote will not, however, fully explain how to design with and analyze instrumentation amplifiers.

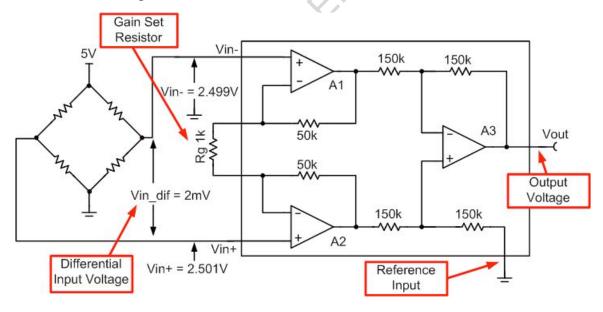


Fig. 10.1: Overview Of Three-Amp Instrumentation Amplifier

A sensor such as a resistive bridge generates the input signal for an INA. In order to understand the gain equations for an INA, you must first understand the formal definition of the common mode and differential components in the input signal. The common-mode signal is the average signal on both inputs of the INA. The differential signal is the difference between the two signals. So, by definition, half of the differential signal is

above the common-mode voltage, and half of the differential signal is below the common-mode voltage. The signal sources in Fig. 10.2 represent the definition of common-mode and differential signals.

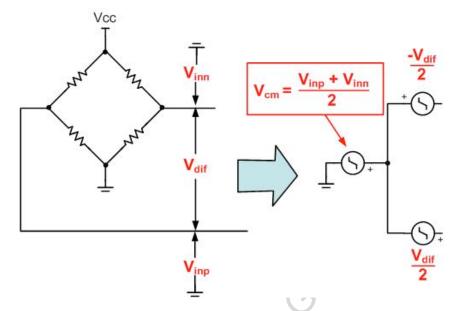


Fig. 10.2: Definition Of Common-Mode/Differential Signals

Now we will apply the signal-source representation of the common-mode and differential voltage developed in Fig. 10.2 to a three-amp INA and solve for the gain equation. This exercise gives valuable insight and intuition into our noise analysis. We will simplify the analysis by separating the input stage from the output stage (see Fig.10.3) allowing us to analyze each separately so that we may combine them later to achieve the total result.

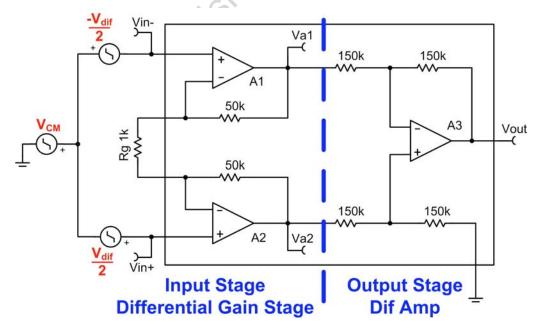


Fig. 10.3: Initiating Three-Amp INA Analysis

In Fig. 10.4 we begin the analysis by using symmetry to split the upper and lower halves of the input stage. Each half of the amplifier can be seen as a simple, non-inverting amplifier (with Gain = Rf/Rin +1). Note that the gain set resistor is also split in half, so the gain of each half is: Gain = 2Rf/Rg +1. Note the transfer of the common-mode voltage (Vcm) to the output of both halves of the amplifier.

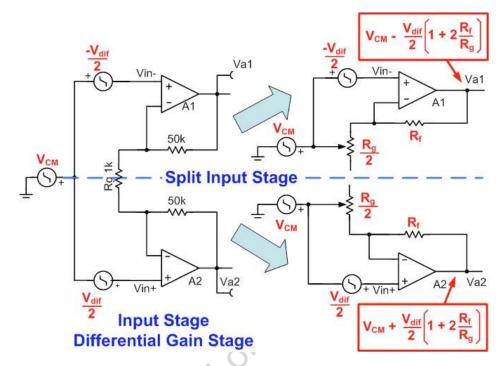


Figure 10.4: Analysis Of Three-Amp INA Input Stage

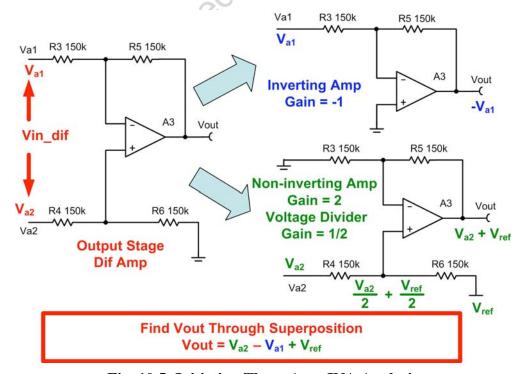


Fig. 10.5: Initiating Three-Amp INA Analysis

Fig. 10.5 shows the analysis of the INA output stage. This amplifier topology is commonly referred to as a differential amplifier (diff-amp). To analyze the output stage, we break the amplifier in half, analyze both sections, and use superposition to combine the results. The top half of the amplifier is a simple inverting amplifier with a gain of negative one: Vout = -Vin.

The bottom half of the amplifier is a non-inverting amplifier with a voltage divider connected to the inputs. Note that the bottom half of the amplifier has two inputs. One input (Va1) is from the input stage, and the other input (Vref) is from the reference pin. The voltage divider (R4 and R6) divides both inputs by two. The gain of the non-inverting amplifier is two: R5/R4 + 1. The total gain seen by Va2 and Vref is one: divider gain x non-inverting gain = $0.5 \times 2 = 1$.

Combining the result from both halves of the amplifier yields the diff-amp's equation: Vout = Va2 - Va1 + Vref. Next combine the results from Figs. 10.4 and 10.5 for the final transfer function. Note that all the gain is in the first stage; the second stage converts the differential output of the first stage to a single-ended signal. The reference voltage adds directly to the output (gain for the reference signal = 1).

$$\begin{split} & \mathbb{V}_{a1} = \mathbb{V}_{cm} - \frac{\mathbb{V}_{dif}}{2} \cdot \left(1 + 2\frac{R_f}{R_g}\right) & \text{[1] Input Stage} \\ & \mathbb{V}_{a2} = \mathbb{V}_{cm} + \frac{\mathbb{V}_{dif}}{2} \cdot \left(1 + 2\frac{R_f}{R_g}\right) & \text{[2] Input Stage} \\ & \mathbb{Bottom Half} \\ & \mathbb{V}_{out} = \mathbb{V}_{a2} - \mathbb{V}_{a1} + \mathbb{V}_{ref} & \text{[3] Output Stage} \\ & \mathbb{V}_{out} = \left[\mathbb{V}_{cm} + \frac{\mathbb{V}_{dif}}{2} \cdot \left(1 + 2\frac{R_f}{R_g}\right)\right] - \left[\mathbb{V}_{cm} - \frac{\mathbb{V}_{dif}}{2} \cdot \left(1 + 2\frac{R_f}{R_g}\right)\right] + \mathbb{V}_{ref} & \text{[1] and [2]} \\ & \mathbb{V}_{out} = \mathbb{V}_{dif} \left(1 + 2\frac{R_f}{R_g}\right) + \mathbb{V}_{ref} & \text{[4] Simplify} \end{split}$$

Fig. 10.6: Initiating Three-Amp INA Analysis

Noise Model Of Three Amp-Instrumentation Amplifier

Fig. 10.7 shows the op amp noise sources included in the INA schematic. Note that each resistor also has a thermal noise associated with it. You can lump all of these noise sources into a single noise source at the input of the INA, or as two noise sources for the input and output stage of the INA. Fig. 10.8 shows the simplified noise models with one or two noise sources. Part II of this series introduces the noise model and Part IV shows a simplified version for use in Spice analysis.

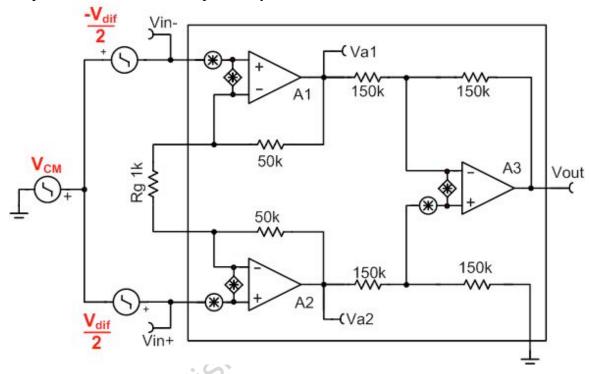


Fig. 10.7: Noise Model For Three-Amp INA

The two-stage model (at the top of Fig. 10.8) has a voltage noise source for the input stage (Vn_in) and a voltage noise source for the output stage (Vn_out). Vn_RTO (voltage noise referred to the output) identifies the output noise for the entire INA. To compute the noise Vn_RTO, add the root sum of the square of the input noise times the gain and the output noise (see Equation 5 in Fig. 10.8). To compute the noise referred to the input, divide the output noise by the INA Gain (see Equation 6 in Fig. 10.8).

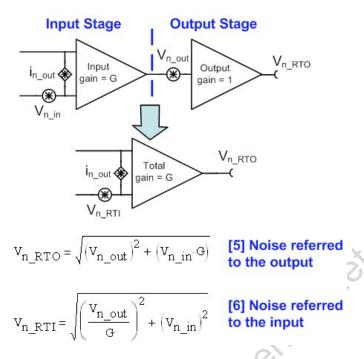
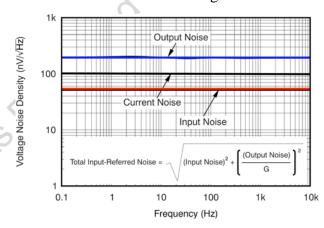


Figure 10.8: Simplified noise model for one or two stages.

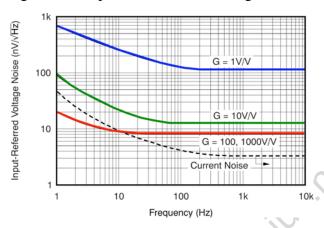
Fig. 10.9 shows the spectral density curve for the INA333. Note that the figure shows two separate curves for the input-stage noise and the output-stage noise. To use this curve, combine the input and output stage noises using Equations 5 or 6. Equation 5 is included in the spectral density curve. The table shows how the input-stage noise dominates at high gain (eg input referred noise is 50 nV/rtHz for gains of 100 and 1000).



G	Total Input-Referred Noise (nV/rtHz) [Equation 6]	Total Output Noise (nV/rtHz) [Equation 5]
1	206.2	206.2
2	111.8	223.6
5	64	320
10	53.9	539
100	50	5000
1000	50	50,000

Fig. 10.9: Noise Model For Three-Amp INA333

Some spectral density curves combine the input-stage and output-stage noise into a single curve. The INA128 noise spectral density curve shown in Fig. 10.10 combines the noise for input and output stages into a single curve. Note that there are several curves at different gains. For low gains, both the input- and output-stage noise is significant (gains of 1 to 10). For higher gains, the input noise is dominant (gains of 100 through 1000).



G	Total Input-Referred Noise (nV/rtHz) Taken directly from graph	Total Output Noise (nV/rtHz) Input x Gain
1	110	110
10	12	120
100	8	800
1000	8	8000

Fig. 10.10: Noise Model For Three-Amp INA128

People sometimes look at the spectral density (Fig. 10.10, again) and mistakenly conclude that output noise decreases with gain. Output noise will always increase with larger gain. The correct conclusion is that both the input and output stages contribute noise at low gains, but the input dominates at high gains. Since noise is typically a concern in high gains, IC designers optimize the input stage for low noise. It is not as important for the output-noise stage to be low because the input stage normally dominates. IC designers do not optimize the output stage noise performance in order to keep the amplifier's quiescent current as low as possible (recall from Part VII that noise is inversely proportional to the quiescent current of the amplifier).

Hand Analysis of Three-Amp Instrumentation Amplifier

In this section, we will compute the expected output noise for a typical INA application. The best way to do this is to analyze the different sections of the circuit separately and combine the results. This analysis will show us which noise sources are significant and which noise sources we can neglect. The ability to determine the dominant noise source is critical in designing a low-noise system; this can save the effort of attempting to reduce noise in an element that does not have a significant effect on noise performance.

Fig. 10.11 shows the example circuit that we will analyze. The gain of this circuit is 101 (Gain = 1 + 100 k/1 k). The circuit uses a single-supply, 5 V-instrumentation amplifier. Here we use a reference buffer to drive the reference pin to half the power supply. This allows the output to swing symmetrically with bipolar input signals. The reference buffer is necessary because the reference pin is relatively high impedance and any series resistance can create a voltage-divider error at the non-inverting input of A3 (see Reference [1]). The input to this circuit is a bridge sensor which can measure a wide range of different signals (eg pressure, strain, acceleration, etc). For the purpose of this analysis, however, we model the bridge sensor simply as four resistors.

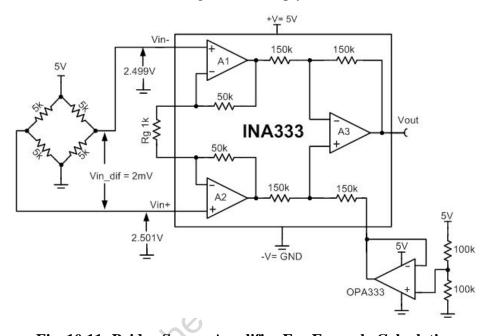


Fig. 10.11: Bridge Sensor Amplifier For Example Calculation

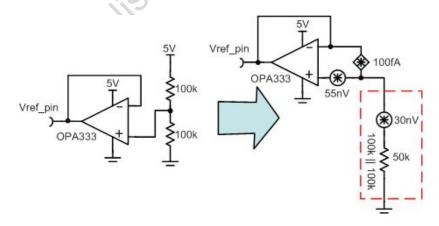


Fig. 10.12: Noise Equivalent Circuit For Reference Buffer Circuit

Fig. 10.12 shows how to compute the reference buffer noise output. The buffer has a voltage divider of two $100~\text{k}\Omega$ resistors which are in parallel from a noise point of view (ie the 5 V supply is at ac ground). For the total noise we consider the divider thermal noise, the voltage noise developed from the divider current, and the op amp noise.

Fig. 10.13 shows the calculations for the reference buffer (from Fig. 10.12). First, we compute the thermal noise from the voltage divider (28.7 nV/rtHz). The voltage noise from the current noise multiplied by the voltage divider resistance is very small (5 nV/rtHz). Typically, you can neglect current noise from MOSFET op amps unless the input resistance is very large (eg greater then $10 \text{ M}\Omega$). The op amp noise dominates the total noise of the reference buffer (62.2 nV/rtHz).

One design consideration for the reference buffer is the thermal noise of the resistor divider. Replacing the $100~\text{k}\Omega$ resistors with $10~\text{k}\Omega$ resistors can significantly reduce thermal noise. Making this change reduces the output noise to 55~nV/rtHz. The only way to reduce the noise further is to change the op amp. Depending on the application, these changes may not be practical. In this example, the OPA333 and the voltage divider don't consume much power. Changing the voltage divider and op amp to improve noise performance can increase power consumption significantly. Furthermore, we will later see that the noise contribution of the reference buffer is not significant in this circuit.

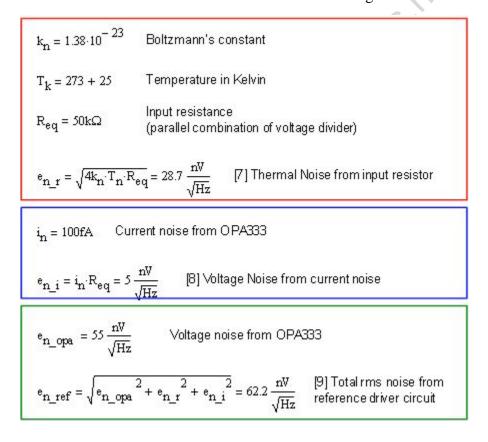


Fig. 10.13: Reference Buffer Noise Calculations

Fig. 10.14 illustrates how to analyze the thermal noise of the resistive bridge sensor and the effect of the current noise. The easiest way to perform this analysis is to consider each input separately and use superposition to combine the noise from each input. Vcc acts as an ac ground, so each input sees two legs of the bridge resistance in parallel. Thus, each input sees one-half of the bridge resistance (R/2) as an equivalent input resistance.

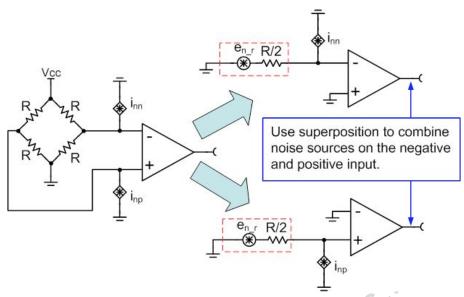


Fig. 10.14: Noise Model Of Bridge/Input Current Noises

$$\begin{split} &i_{nn}\cdot\frac{R}{2} &\quad \text{Voltage noise from current noise} \\ &e_{n_rb} = \sqrt{4k_n\cdot T_n\cdot\frac{R}{2}} &\quad [10] \, \text{Resistor} \\ &\text{Noise} \\ &\text{Use superposition to add the noise from the input resistance and both current noise sources} \\ &e_{in_i} = \sqrt{\left(i_{nn}\cdot\frac{R}{2}\right)^2 + \left(e_{n_rb}\right)^2 + \left(i_{np}\cdot\frac{R}{2}\right)^2 + \left(e_{n_rb}\right)^2} \\ &\text{Assume } |i_{nn}| = |i_{np}| \\ &\text{Note that these sources are uncorrelated} \\ &e_{in_i} = \sqrt{2\left(i_n\cdot\frac{R}{2}\right)^2 + 2\left(e_{n_rb}\right)^2} \quad [11] \, \text{Total Noise from input resistors and current source} \end{split}$$

For this example (R=5k
$$\Omega$$
, in = 100fA/rtHz)
$$e_{n_rb} = 6.4 \frac{nV}{\sqrt{Hz}} \qquad \text{Resistor noise}$$

$$i_{nn} \cdot \frac{R}{2} = 0.25 \frac{nV}{\sqrt{Hz}} \qquad \text{Voltage noise from current noise}$$

$$e_{in_i} = \sqrt{2(0.5)^2 + 2(9.1)^2} = 9.1 \frac{nV}{\sqrt{Hz}} \qquad \text{Total Noise from input resistors and current source}$$

Fig. 10.15: Thermal/Current Noise Calculations

Fig. 10.15 shows the calculations for the thermal noise from the bridge sensor and effect of current noise. The current noise is extremely low in this example because the INA333 is a MOSFET instrumentation amplifier (100 fA/rtHz). We multiply the current noise by the equivalent input resistance to convert it to a voltage noise.

Fig. 10.16 shows all the noise components in our example circuit. Note that the input stage and sensor noise add as the root sum of the squares (RSS). Now multiply the combined input and sensor noise by the INA gain and add RSS with the output stage noise and reference buffer noise.

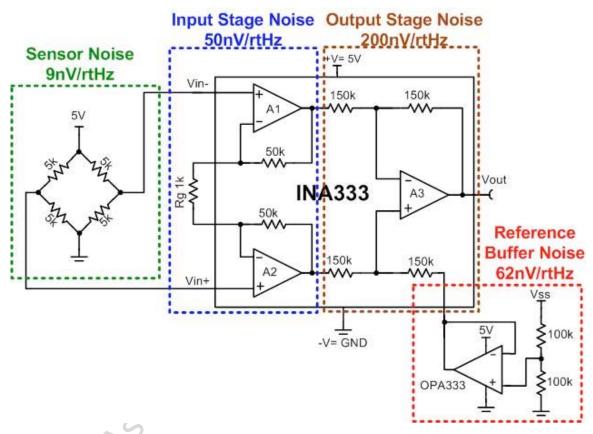


Fig. 10.16: Summary Of Noise Components In Example Circuit

When conducting noise analysis, you often have a single dominant source of noise and can neglect all other noise sources without introducing significant error. A good rule of thumb is that a noise source is dominant if it is three times larger then the other sources. Keep in mind to square the factor of three because you add noise as the root sum of the squares.

Fig. 10.17 illustrates the rule of three.

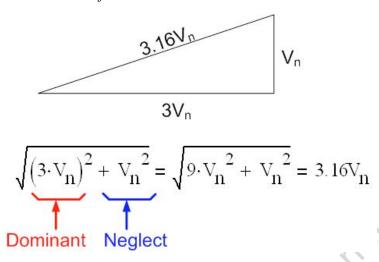


Fig. 10.17: Rule Of Three Identifies Dominant Noise Components

For our example circuit in Fig. 10.11, you can see that the input stage noise dominates. Remember to use the rule of three when comparing the different components with each other. The input stage noise is 50 nV/rtHz and the total combined noise referred to the input is 50.8 nV/rtHz. It is important to identify the dominant noise components so that you don't waste time trying to optimize circuit noise that does not matter. In this example the only important factor to overall noise is the input-stage noise. Thus, optimizing the reference buffer noise will not yield any measurable benefit.

[12] Noise spectral density refered to the input for this example

Noise_Spec_Den_RTI =
$$\sqrt{V_{n_in_stage}}^2 + V_{n_bridge}^2 + \left(\frac{V_{n_out_stage}}{G}\right)^2 + \left(\frac{V_{n_ref_buf}}{G}\right)^2$$

Noise_Spec_Den_RTI = $\sqrt{(50)^2 + (9)^2 + \left(\frac{200}{100}\right)^2 + \left(\frac{62}{100}\right)^2} = 50.847 \frac{\text{nV}}{\sqrt{\text{Hz}}}$

Dominant Neglect Approximately equal to the dominant term

Fig. 10.18: Input Noise Dominates In This Example

The next step in this analysis is to compute the total rms noise and peak-to-peak noise at the output. In order to compute rms noise we must know the noise bandwidth. You can use the gain vs frequency plot from the data sheet to estimate the order of roll-off of high frequency gain.

Fig. 10.19 illustrates how gain rolls off at -20 dB/decade (equivalent to a first-order or single-pole filter). We use the roll-off to compute the brick wall correction factor (K_n), and compute the noise bandwidth by multiplying the brick wall correction factor by the 3 dB bandwidth. In this example, Kn = 1.57 because the INA333 high frequency roll-off is single pole. You can read the 3 dB bandwidth directly from the "Frequency Response" table in the <u>data sheet</u> (3.5 kHz). Thus, for this example, the noise bandwidth is $BWn = 1.57 \times (3.5 \text{ kHz}) = 5.495 \text{ kHz}$. Part II of this TechNote series discusses the brick wall correction factor and noise bandwidth.

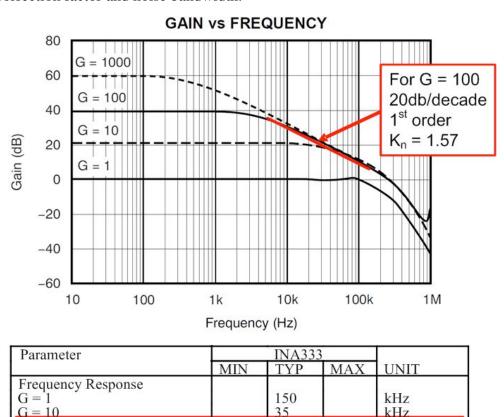


Fig. 10.19: Noise Model For Three-Amp INA

3.5

350

kHz

G = 100

= 1000

Fig. 10.20 shows the final calculations for the example circuit. Note that Part I of this TechNote series describes the method used to compute total noise. This example uses a chopper-stabilized instrumentation amplifier. Thus, the amplifier does not have any 1/f noise and the calculations are simpler. We calculate the rms noise by taking the square root of the noise bandwidth multiplied by the input noise (see Equation 14, Fig. 10.20), then approximate the peak-to-peak voltage using six times the rms value (see Equation 15, Fig. 21).

G = 100	INA Gain
$V_{in_RTI} = 50.85 nV / ttHz$	From "Input referred noise" equation
$f_{H} = 3.5kHz$	From data sheet table for gain = 100
$K_n = 1.57$	For first order function See Gain vs Frequency in the data sheet
$BW_{\mathbf{n}} = \mathbf{f}_{\mathbf{H}} \cdot \mathbf{K}_{\mathbf{n}} = 5.495 \text{kHz}$	Noise Bandwidth
$e_{n_out} = G \cdot V_{in_RTI} \sqrt{BW_n} = 376.9 \mu V \text{ rms}$	[13] RMS Output Noise
$e_{n_outPP} = 6.e_{n_out} = 2.26 mVpp$	[14] Peak-to-Peak Output

Fig. 10.20: Final Noise Calculation For Bridge Resistor Circuit

Simulation of Three-Amp Instrumentation Amplifier

Any SPICE simulator can simulate the circuit from Fig. 10.11. Texas Instruments offers downloads of <u>TINA SPICE</u> and the INA333 model at no cost. The INA333 model correctly models noise and most other parameters of interest. <u>Part IV</u> of this TechNote series shows how to simulate noise in op amp circuits using the same methods we use for INAs.

Fig. 10.21 shows the "noise analysis" options available from TINA SPICE. When the "Output Noise" box is checked, the simulator will create a spectral density plot at each test point in the schematic. When the "Total Noise" box is checked, the simulator will generate an rms noise plot (ie the power spectral density plot integrated).

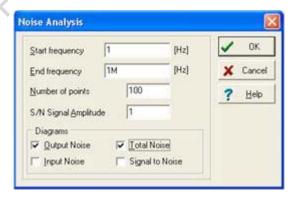


Fig. 10.21: TINA Noise Analysis Options

Fig. 10.22 shows the simulated spectral density plot for the example circuit as generated in TINA SPICE using the "Output Noise" option. Fig. 10.23 shows the rms noise (square root of power spectral density integrated).

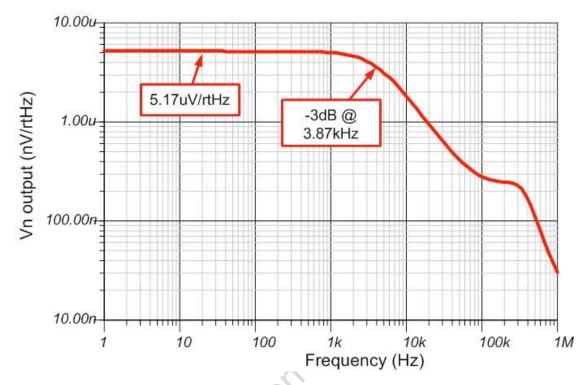


Fig. 10.22: Spectral Density At Output Of INA333 Example Circuit

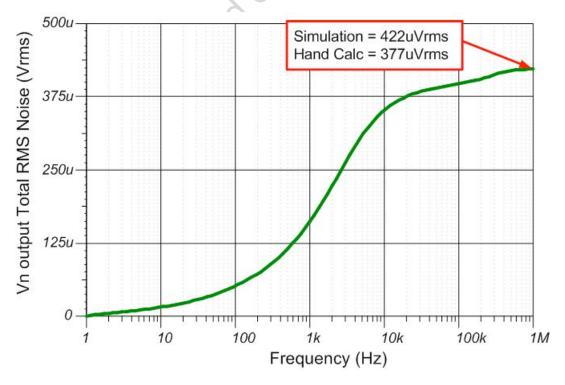


Fig. 10.23: Rms Output Noise For INA333 Example Circuit

Note that the simulated and calculated rms noise in Fig. 10.23 do not match exactly (simulated = 422 μ Vrms, calculated = 377 μ Vrms). There are several reasons for this minor discrepancy. First, the spectral density of the simulated result is slightly higher then the calculated spectral density. Second, the bandwidth for the simulated result is slightly wider then the bandwidth from the data sheet. Finally, the simulator gain roll-off has a "bump" at approximately 200 kHz. The hand calculations did not account for this bump. Fig. 10.24 illustrates these discrepancies.

Engineers are often concerned when they see discrepancies between simulations and hand calculations. In this case, the error is approximately 10% which, on noise calculations, is not very large. Remember that the datasheet gives the specifications as typical, so you will see variations greater then 10% for real world devices. In this example, the hand calculations are more accurate than the simulated results for bandwidth and spectral density below 1 kHz because they derive directly from the data sheet. The simulation more accurately follows the data sheet at the 200 kHz-bump in the spectral density curve. In general, you should ignore discrepancies on this order.

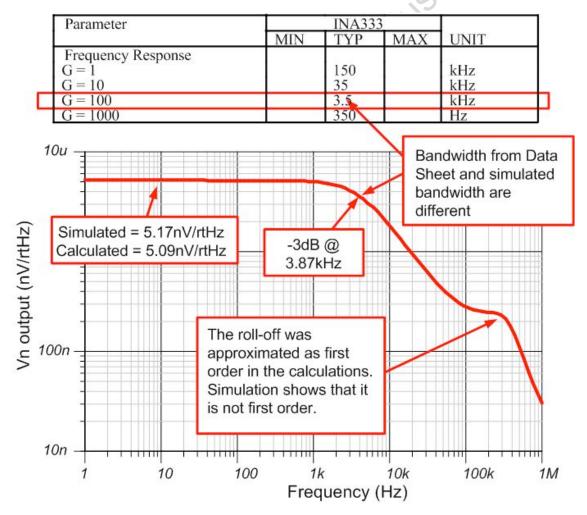


Fig. 10.24: Simulated Versus Hand Calculations

Reducing Noise With Averaging Circuit

One way to reduce noise is to connect several amplifier inputs together and average the outputs using an op amp. Fig. 10.25 shows an inverting-averaging circuit. All of the input resistors (R1, R2, R3, ...RN) must be equal for the averaging feature to operate. Additionally, the feedback resistor (Rf) must be equal to the input resistor value divided by the number of input resistors.

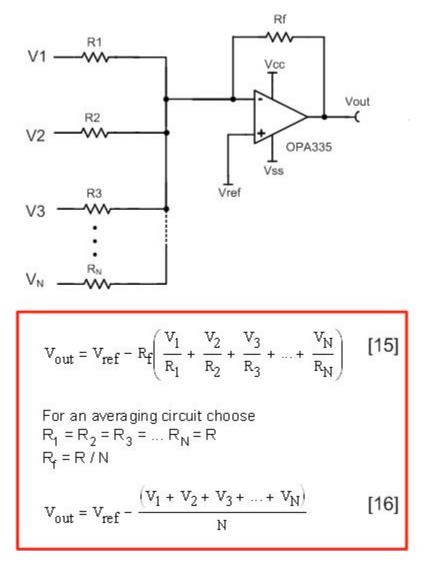


Fig. 10.25: Op Amp Averaging Circuit

Fig. 10.26 shows mathematically how the averaging circuit reduces noise. The averaging effect assumes that the noise sources are equal and uncorrelated. The output noise is the input noise divided by the square root of the number of averages.

$$v_{\text{noise_output}} \ = \sqrt{\left(\frac{v_{\text{noise1}}}{N}\right)^2 + \left(\frac{v_{\text{noise2}}}{N}\right)^2 + \left(\frac{v_{\text{noise3}}}{N}\right)^2 + \ldots + \left(\frac{v_{\text{noiseN}}}{N}\right)^2}$$

Where v_{noise1} , v_{noise2} , v_{noise3} ,... v_{noiseN} are noise sources

If you assume that v_{noise1}, v_{noise2}, v_{noise3}, ... v_{noiseN} are equal uncorrelated noise sources, then

$$v_{\text{noise_output}} = \sqrt{N \left(\frac{v_{\text{noise}}}{N}\right)^2} = \sqrt{\frac{v_{\text{noise}}^2}{N}} = \frac{v_{\text{noise}}}{\sqrt{N}}$$
 [17]

Fig. 10.26: Derivation Of Output Noise For Averaging Circuit

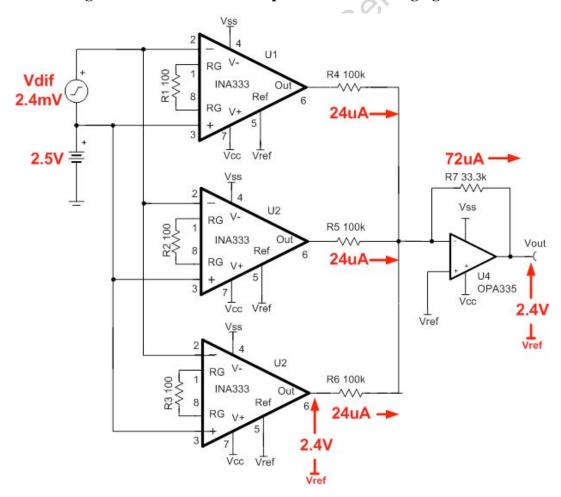


Fig. 10.27: Three-Amp Averaging Circuit For INA333

Fig. 10.27 shows a practical circuit averaging three INA333 amplifiers. Note that the INA333 gain is set to 1001 in this example (Gain = 1 + 100 k/100). The figure also shows how the three amplifier inputs connect together and how the three INA333 outputs connect to the average circuit. This circuit effectively acts as a single INA333 with noise reduced by the factor of 1/rt(3). Note that the current noise from the INA333 input amplifiers adds (root sum of squares). Thus, while this circuit is an effective way to decrease voltage noise it also increases current noise.

Select R4, R5, and R5 to limit current to a reasonable value. The INA333 and OPA335 are low-power devices and care must be taken to prevent excessive current. Engineers sometimes choose a smaller value for averaging input resistors to minimize noise but this will not affect the overall noise because the INA333 noise dominates (50 nV/rtHz x 1000 = $50 \,\mu$ V/rtHz). Select R7 to scale the gain for averaging (eg $100 \, k / 3 = 33 \, k$).

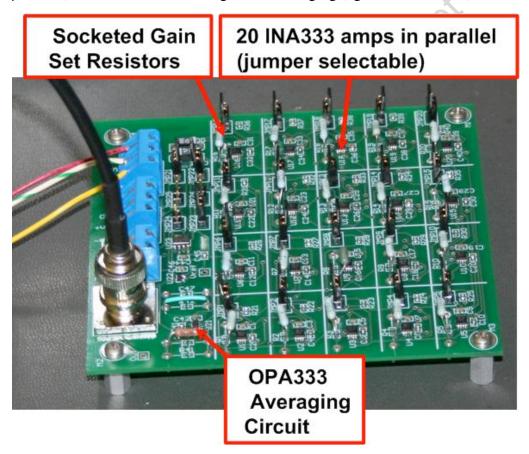


Fig. 10.28: Circuit Used To Demonstrate Effect Of Averaging On Noise

Fig. 10.28 shows the test board used to prove the effectiveness of the averaging circuit. The number of amplifiers you can average is jumper selectable. The instrumentation amplifier gain is set using socketed through-hole resistors. The goal was to make the board flexible enough to do a wide range of experiments. Using the jumpers and through-hole components make the size of the PCB much larger than it would be with only surface-mount components. The size is important because the longer traces are more likely to pick up extrinsic noise (eg 60 Hz, RFI interference, etc).

Fig. 10.29 shows the total output noise versus the number of amplifiers averaged. The noise decreases according to the equation $Vn_output = Vnoise / rt(n)$. Note that a significant noise reduction is made using a four-amplifier circuit (four amplifiers reduces noise by one-half $(1 / rt (4) = \frac{1}{2})$). Using a larger number of amplifiers further reduces noise, but this may not be practical because of PCB area, cost, and complexity.

Noise vs Number of Amplifiers

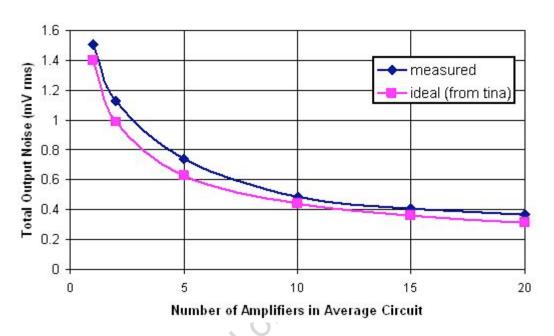


Fig. 10.29: Noise Versus Number Of Amplifiers For INA333

Fig. 10.30 (overleaf) shows the measured and simulated spectral density for the INA333 averaging circuit. The measured and simulated results compare closely with each other.

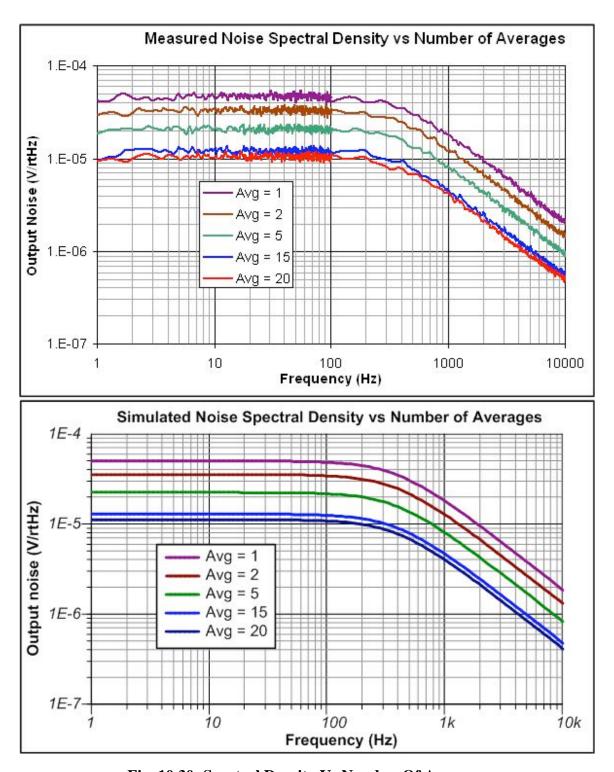


Fig. 10.30: Spectral Density Vs Number Of Averages

Summary

In this TechNote, we have discussed how to calculate and simulate instrumentation amplifier noise. We have also looked at using an averaging circuit to create a composite, low-noise amplifier.

Acknowledgments

Special thanks to the following individuals from Texas Instruments for their technical insights:

- Rod Burt, Senior Analog IC Design Manager
- Matt Hann, Senior Applications Engineer
- Tim Green, Applications Engineering Manager

References

[1] Hann, Gina, "Selecting the right op amp," *Electronic Products Magazine*, November 21, 2008: http://www2.electronicproducts.com/Selecting the right op amp-article-facntexas nov2008-html.aspx.

About the Author

Arthur Kay is a Senior Applications Engineer at Texas Instruments where he specializes in the support of sensor signal conditioning devices. Prior to TI's acquisition, he was a semiconductor test engineer for Burr-Brown and Northrop Grumman Corp. Art graduated from Georgia Institute of Technology earning an MSEE. He can be reached at ti artkay@list.ti.com.

